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The undetected propagation of a fatigue crack constitutes a significant cause of aircraft and other structural failures. To raise the structural failure load to a relatively high level, the manufacturer can divide the structure into many small elements, which significantly increases the ability of a structure to tolerate an element failure. This report presents a procedure for calculating the probability that the element has not failed, as a function of the crack propagation time and hence the crack's length. The procedure's form is so simple that computations with a desk calculator can yield reasonably accurate results. To illustrate this, the report uses data that an aircraft manufacturer developed for the structural components/elements that currently limit the service life of an existing transport aircraft. Refs. (BG)

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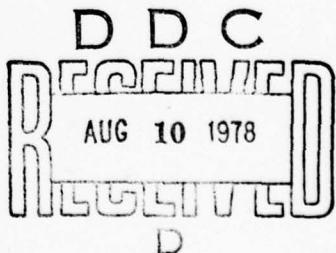
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**R-2238-RC**

June 1978

# Probability That the Propagation of an Undetected Fatigue Crack Will Not Cause a Structural Failure

J. R. Gebman, P. C. Paris



PREFACE

This report documents research conducted as a public service by Rand using its own funds. The report draws upon recent Rand research conducted for and reported to the Air Force. That research has revealed several methodological deficiencies in the current engineering procedures for evaluating the structural modifications required to extend an airframe's service life. To help fill this gap in engineering analysis procedures, Rand initiated a Fatigue Failure Analysis project. This report documents the results of that project.

The findings discussed here should interest organizations that manufacture, operate, or regulate aircraft and other structures with elements that are prone to fatigue failure. The findings should particularly interest organizations responsible for transport aircraft, because a transport serves as the basis for the example application. The Federal Aviation Administration (FAA) and the U.S. Air Force may wish to consider encouraging the use of this approach to enhance the fail safety features of new designs, and to improve the effectiveness of maintenance actions intended to reduce the chances of inflight structural failures. Recently, the FAA proposed a rule change that should allow manufacturers of commercial transports to use this approach.<sup>1</sup>

Professor Paul Paris is a consultant to Rand's Engineering and Applied Sciences Department.

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<sup>1</sup>"Transport Category Airplane Fatigue Regulatory Review Program, Fatigue Proposals," Federal Aviation Administration, 14 CFR, Part 25, Docket No. 16280; Notice No. 77-15, appeared in *Federal Register*, Vol. 42, No. 157, August 15, 1977.

SUMMARY

The undetected propagation of a fatigue crack constitutes a significant cause of aircraft and other structural failures. Although fracture mechanics methods can generally predict how fast a given crack will propagate through an element in a metal airframe, so far no method can adequately predict when cracks will start because of design, manufacturing, maintenance, or operational anomalies. Moreover, in many relevant situations, reliable information does not exist for the probability that inspection procedures will locate a crack. When such detection does not occur, an applied load eventually causes the cracked element to fail, and if large enough, also causes the structure to collapse. To raise this structural failure load to a relatively high level, the manufacturer can divide the structure into many small elements, which significantly increases the ability of a structure to tolerate an element failure. Until now, however, a credible and generally meaningful procedure for assessing this benefit has not existed. This report presents the basic equations for such a procedure.

The proposed procedure calculates the probability that the undetected propagation of a given fatigue crack in an element will not lead to a structural failure. It also provides the probability that the element has not failed, as a function of the crack propagation time and hence the crack's length. It requires information about:

- o The time-varying distribution of applied loads,
- o How fast cumulative operations cause the crack's size to enlarge,
- o How the crack's size influences the element's remaining strength, and
- o The applied load required to fail the structure once the cracked element has failed.

This procedure does not use estimates of how and when a crack starts. Nor does it require the definition of a specific maximum load

condition that a fatigue damaged structure must tolerate. However, it does require the assumption that an element failure will not go unnoticed if the aircraft survives the flight during which the element fails.

The procedure has a form so simple that computations with a desk calculator can yield reasonably accurate results. To illustrate this, the report uses data that an aircraft manufacturer developed for the structural components/elements that currently limit the service life of an existing transport aircraft. To be safe, the operator imposed the current service limit based on a reasonably conservative assumption that a crack as large as .05 inch may have existed in a wing component during fabrication. Based on the manufacturer's fracture mechanics calculations, the operator estimated that it should take only about 8000 flight hours for such a crack to propagate to a length where a limit load would cause a component failure. In the event that two adjoining components failed because the adjoining component also contained a crack, the operator surmised that the structure would probably fail because the manufacturer had not *intentionally* designed the structure to withstand a dual component failure. Thus, the operator imposed the current limit of 8000 hours. The example calculation yields a 97.4 percent probability that even such a dual component failure should *not* cause a structural failure. Moreover, if the adjoining component does not contain a significant strength degrading crack, the probability increases to 99.9 percent because the stresses near the failure site have a 30 percent lower intensity with the single component failure than with the double component failure.

This calculated capability to tolerate single and dual component failures does not necessarily extend to the other types of aircraft that the operator uses. For example, compared with the example aircraft, some of the other aircraft experience higher stress levels and have far fewer components comprising the structure of the wing's lower surface. Moreover, whereas some aircraft have significant load-carrying components buried inside the wing's lower surface, this does not occur with the example aircraft. Thus, because the manufacturer of the example aircraft has divided the lower surface structure into eleven distinct

components, each clearly visible from outside the aircraft, that aircraft has a significant capability to survive either a single or dual component failure.

An application of this calculation procedure to both the single component and the dual component failure scenarios for other aircraft should provide the operator with new and significant comparative insights into the priorities for structural modifications designed to extend the service lives of various aircraft.

The example also shows that 98 percent of the component failures considered in the example aircraft should occur after the postulated crack has grown from underneath a lap joint. Although this should significantly increase the chances that detection of the crack might avert a component failure, the previously stated probabilities have not accounted for such a possibility.

The procedure generally does not apply to situations where a large number of strength degrading fatigue cracks have developed over a wide area, or prompt detection of a failed element would not occur, or the nature of the structure or its operating environment makes the preparation of the inputs impractical.

For many situations, however, the procedure should help manufacturers and operators to more systematically identify and prioritize failure critical components. Thus, we recommend that manufacturers and operators adopt such a probabilistic procedure to

1. Help evaluate the need for major structural modification programs.
2. Augment current service life assessment procedures.
3. Inform management about a structure's damage tolerance capability throughout the structure's life cycle--from initial design through final retirement.

ACKNOWLEDGMENTS

J. R. Rice originally discussed the essential aspects of this approach in an unpublished paper he prepared for Professor Paris. W. L. Stanley, K. M. Bloomberg, and P. K. Dey helped to develop a computer program that was used to explore earlier versions of the approach. M. L. Juncosa, M. P. Kaplan, G. K. Smith, and K. A. Solomon provided helpful comments on earlier drafts.

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SYMBOLS

$a(t)$  = crack length at time  $t$ ; in inches.

$f$  = factor including the influence of second order terms.

$i, j, k$  = indexes identifying operating intervals.

$n(\sigma)$  = number of times that the maximum load intensity per operating interval exceeds  $\sigma$  during one design lifetime containing  $t_d$  flying hours.

$P_{ef}(t_0, t_i) = 1 - P_{nef}(t_0, t_i)$ .

$P_{nef}(t_0, t_i)$  = probability that the postulated crack condition does not fail the element at any time during the interval  $(t_0, t_i)$ .

$P_{nsf}(t_0, t_i)$  = probability that the postulated crack condition does not contribute to a structural failure at any time during the interval  $(t_0, t_i)$ .

$P_{sf}(t_0, t_i) = 1 - P_{nsf}(t_0, t_i)$ .

$r_{ef}(t)$  = expected rate at which the maximum load intensity per operating interval exceeds the element failure stress  $(\sigma_{ef})$ ; occurrences per unit of operating time at time  $t$ .

$r_{sf}(t)$  = expected rate at which the maximum load intensity per operating interval exceeds the structural failure stress  $(\sigma_{sf})$ ; occurrences per unit of operating time at time  $t$ .

$t$  = cumulative operating time measured in flying hours.

$t_i$  = time when the  $i$ th operating interval ends.

$t_d$  = design lifetime; basis for the function  $n(\sigma)$ .

$\bar{t}_\alpha$  = midpoint of calculation interval  $\alpha$ .

$\Delta t$  = length of the operating interval.

$\tilde{t}$  = time when the minimum load intensity level that can fail an element equals the residual strength of the structure once the element has failed  $(\sigma_{rs})$ .

$x_i$  = arbitrary function.

$\alpha, \beta$  = indexes identifying calculation intervals.

$\sigma$  = intensity of the internal structural loads that the applied loads would cause in an idealized structure that does not contain any holes, cutouts, or cracks (e.g., gross area stress); measured at the location of the postulated crack; in ksi (1 ksi = 1000 psi).

$\hat{\sigma}(t_i)$  = maximum load intensity for the  $i$ th operating interval.

$\sigma_{ef}(a)$  = minimum load intensity level at which a crack of length  $a$  can fail a given element.

$\sigma_{sf}(a)$  = minimum load intensity level at which a crack of length  $a$  can contribute to a structural failure.

$\sigma_{rs}$  = remaining strength of the structure when the element containing the postulated crack fails.

$\tau$  = dummy variable for the time  $t$ .

I. INTRODUCTION

The strength capability of an aircraft structure frequently declines with service use because the repeated loading and unloading of the structure causes the spawning and enlargement of fatigue cracks that progressively weaken the structure. If undetected, a fatigue crack will usually enlarge to a size where the loads applied during a flight will cause the element containing the crack to fail. If adjoining elements cannot pick up the internal structural loads carried by the failed element and transmit them around the failure site, the failure advances into these elements as well, and the total load that the remaining unfailed elements must pick up and redistribute around the failure site is increased. If the remaining elements cannot stop the progression of the failure, this portion of the structure collapses, thereby causing a *structural failure*. Because an accident, usually involving fatalities, occurs when a major portion of an aircraft structure collapses during flight, such an incident is usually referred to as a *catastrophic structural failure*.<sup>1</sup>

This report develops a set of simple formulas for calculating the probability that a structural failure will *not* occur even though a postulated fatigue crack propagates through an element without being detected. The formulas require information on crack propagation and strength degradation, along with a distribution to represent how often the applied structural loads exceed specified levels.

Section II describes the approach, the assumptions, and the inputs. Although the approach is applied chiefly to reducing the chances that an aircraft will experience a catastrophic structural failure, it has potential applicability to other kinds of structures (e.g., pressure vessels). To facilitate its application to related problems, Sec. III provides a general development of the basic equations. Section IV applies the approach to a transport aircraft wing, and Sec. V presents observations and recommendations. Appendixes A through I furnish additional details.

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<sup>1</sup>See Appendix A for some recent examples.

CURRENT APPROACHES

Even though the possibility of a catastrophic structural failure will always exist during every flight of every manned aircraft, the aircraft's manufacturer and operator can in theory use available resources to minimize the probability that it will occur. To accomplish this, two distinct design philosophies have evolved:

1. The *fail-safe* philosophy holds that an aircraft structure should have enough independent elements to provide assurance that for a specified applied load, the failure of any single element will not lead to a catastrophic structural failure.
2. The *safe-life* philosophy holds that the specified operating life for an aircraft structure should not exceed the time required for a specified initial crack to propagate to a length (referred to as the *critical length*) at which a specified applied load would cause the element containing the crack to fail (Refs. 1 and 2).

Although each yields some desirable design features, both philosophies have certain limitations that need not afflict a probabilistic approach.<sup>1</sup> For example, both require the specification of an applied design load. But no matter how large a design load the operator specifies, there is still the possibility that an even larger load may occur. Also, no matter how large an initial crack size the operator specifies for a *safe-life* approach, an even more severe condition may arise.

Aircraft manufacturers and operators currently use fixed extreme values for the design load and the initial crack size. Although such values may reflect the best judgment of the most knowledgeable individuals, neither the manufacturer's nor the operator's management can relate the subjective basis for such assessments to an understanding of the likelihood that a postulated fatigue crack may cause a catastrophic structural failure. Thus managers have difficulty finding a

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<sup>1</sup>See Appendix B for additional discussion.

consistent basis for allocating resources to reduce *uniformly* the chances that a catastrophic structural failure will result from an element failure in an aircraft structure.

#### A PROBABILISTIC APPROACH

This report proposes a probabilistic approach that avoids the extreme value judgments mentioned above and supplements the information provided by the *safe-life* and *fail-safe* approaches. This probabilistic approach, which accepts the existence of the crack as a given and uses a distribution to represent the applied loads, has the following unique features:

- o The approach requires no knowledge of what causes a crack to start or of how fast the crack propagates during its initial phase of development.
- o Once the crack has reached a size where it begins to contribute to the calculated probabilities, the approach accurately represents the crack propagation history calculated by fracture mechanics methods.

#### Potential Application

When this probabilistic approach has been applied to various elements, they can then be ranked according to the likelihood that their failure would lead to a structural failure. The most *failure-critical* elements constitute candidates for either redesign or special maintenance attention, such as more frequent inspections. In some cases, a special rework or even replacement of the element may be warranted.

Such a procedure for consistently quantifying a structure's ability to tolerate undetected fatigue cracks may help manufacturers and operators to apply their available resources more effectively. For example, during the design and manufacturing phase, such a procedure can (1) help identify the most failure-critical elements in a given design; (2) provide a basis for design and manufacturing standards; and (3) give managers an opportunity to see how design tradeoffs may

influence the chances of structural failure. During the operating phase, such a procedure can (1) help reassess the most failure-critical elements based on actual operating conditions; (2) provide a basis for an inspection and modification standard; and (3) give managers/regulators an opportunity to see how inspection and modification trade-offs may influence the chances of structural failure.

Although the approach can be applied to structures other than aircraft, it does not apply to situations where (1) a large number of fatigue cracks may have weakened a structure over a broad general area; or (2) prompt detection of a failed element would not occur; or (3) the nature of the structure or its operating environment makes the preparation of inputs impractical.

#### An Example Application

Say an aircraft operator has established a structural modification schedule based on a postulated crack condition that extends to two overlapping components. He could use the probabilistic approach proposed here to reevaluate the necessity and effectiveness of the proposed modification and to examine how alternative actions could influence the probability of a structural failure. He might consider not making the modification or he might substitute an inspection program. He might also examine components not included in the proposed modification or apply the approach to similar components in other transport aircraft. To illustrate the method's application, this report uses actual data for the same postulated crack condition that the operator has used to set up the modification schedule.

## II. BASIC CONCEPTS, INPUTS, AND ASSUMPTIONS

A human error during design, manufacture, operation, or maintenance, or a mechanical malfunction can create conditions that spawn a fatigue crack in a structural element. Although current engineering procedures cannot reliably predict how often such an event may occur, they can model what happens once a fatigue crack has developed. Unless detected, and depending on the geometry, continuing operation of the structure will cause the crack to enlarge to a size where it eventually weakens the element and causes it to fail. This will not lead to failure of the structure, however, if the following set of conditions is satisfied:

1. The applied loads that occur after the element has failed do not exceed the remaining strength of the structure at the time the element failed.
2. The remaining duration of the flight, during which the element failure occurred, does not provide enough time to spawn an additional fatigue crack that enlarges to a size where it significantly weakens an adjoining element.
3. A fully effective replacement/repair of the failed element occurs before a fatigue crack weakens an adjoining element (e.g., the repair occurs after the flight during which the element failed).

For situations where conditions (2) and (3) hold, this report provides equations for calculating the probability that a postulated crack condition will not lead to a structural failure.

### BASIC CONCEPTS

The *actual probability* that a crack will eventually lead to a structural failure depends on the factors explicitly represented in this approach, as well as on other factors suppressed by assumptions. If these suppressed factors have a negligible influence on the actual

probability, then the *predicted probability* calculated with this method should provide a close approximation to the *actual probability*. Even if some suppressed factors have a significant influence that can be treated separately, the value calculated for the *predicted probability* may still satisfactorily quantify the influence of the factors that the model represents. Although this report refers to the *predicted probability* as simply the *probability*, the user must always ascertain for himself the extent to which this probability approximates the *actual probability*.

The Idealized Representation of the Failure Process

An element will fail when the intensity of the internal structural loads (referred to here as simply *load intensity*,  $\sigma$ ) exceeds the element's remaining strength. But such a failure will not extend to adjoining elements as long as the load intensity does not exceed the remaining strength of the elements adjoining the failure site.

Once a fatigue crack enlarges to a size where it first begins to weaken the element, any additional enlargement reduces the remaining strength of that element. As the remaining strength of an element decreases, the minimum load intensity that can cause the element to fail ( $\sigma_{ef}$ ) decreases (e.g., see Fig. 1). Thus in some situations an element may not fail until the crack is large enough to cause an appreciable degradation in the element's remaining strength. In that case, the load intensity level that causes the element to fail may not exceed the level required to also cause the adjoining elements to fail. In such an event, the *element fails (ef)* but *no structural failure (nsf)* occurs. For such a "safe" (or "noncatastrophic") element failure to occur during a specific operating interval (e.g., during one flight of an aircraft):

1. The element must not have failed during a prior operating interval, and
2. During the designated operating interval
  - a. At some time the load intensity must exceed the minimum level required to fail the cracked element, and

b. At no time following the element failure can the load intensity exceed the minimum level required to fail the remaining structure once the element of interest has failed.

The remaining or residual strength of the structure, once the element has failed ( $\sigma_{rs}$ ), depends on the failure mode postulated for the structure as well as the condition of adjoining elements.

Because the crack of interest will enlarge and the remaining strength of the element containing the crack will decrease during successive operating intervals, our probabilistic approach must explicitly represent the process whereby the crack enlarges as the structure accumulates operating time. It must also account for the

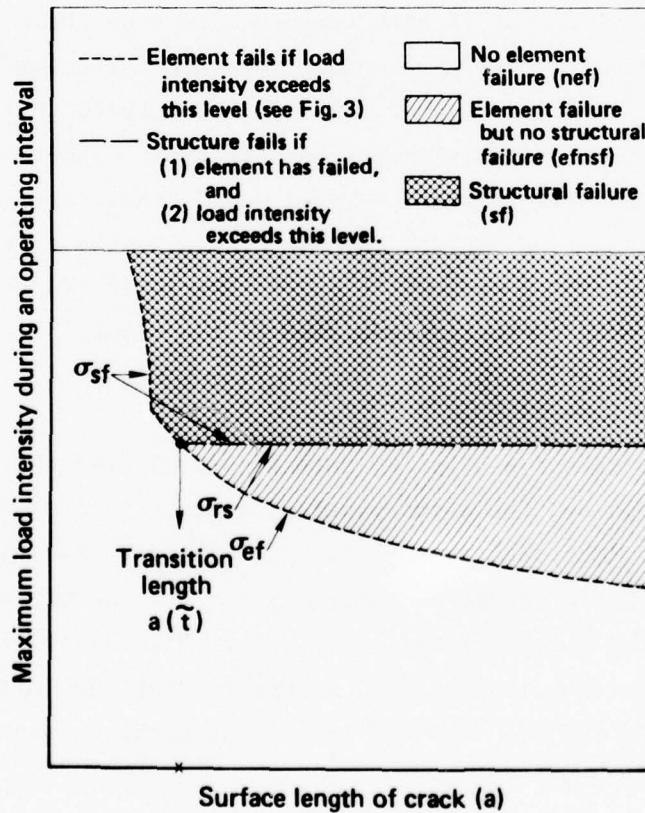


Fig. 1 - Illustration of how crack length and load intensity influence failure events

process whereby crack enlargement reduces the element's remaining strength. Our approach represents both of these processes with deterministic relationships. It also uses a deterministic input to represent the load intensity level at which structural failure can occur, once the element containing the crack has failed. Because load intensity levels vary widely during the operation of such structures--especially aircraft structures--this approach uses a distribution to represent the load intensity.

Definition of an Element

The element of interest is defined as any portion of the structure that has two boundaries that can arrest the propagation of a failure. Thus, the term *element* may refer to one or more structural members. For example, the postulated crack condition might extend to two overlapping structural members at a common fastener hole, where cracks could presumably develop in both members. Because the member with the shorter crack may not have enough remaining strength to confine the failure to the other member, it might be assumed that the failure of one member automatically means that the other member will fail also. In this case, the two members together would constitute the *element* of interest. The *element* concept may also apply to the portion of a panel between two supporting members (e.g., stringers or frames) if those members might stop further propagation of the crack.

DESCRIPTION OF INPUTS<sup>1</sup>

As an illustrative example, this report applies the approach to actual data obtained for a two-panel element that forms part of a wing's lower surface. For this *element of interest*, the calculation procedure requires four basic pieces of information: (1) how often does the load intensity exceed various levels (e.g., Fig. 2); (2) how does the crack length influence the minimum load intensity that can fail the element (e.g., Fig. 3); (3) how does the crack length change over time (e.g.,

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<sup>1</sup> Appendix D discusses how the airframe manufacturer typically prepares the information required by this approach. Appendix E describes how the inputs influence the results.

Fig. 4); and (4) the remaining strength of the structure, once the element of interest has failed (expressed as a load intensity,  $\sigma_{rs}$ ). Because this information can vary for different locations, the discussion in this subsection applies only to the specific crack or cracks that are presumed to exist in the element of interest. The remaining inputs include the average length of the operating interval ( $\Delta t$ ) and the design lifetime ( $t_d$ ) on which Fig. 2 is based.

#### Load Intensity Distribution

The intensity of the internal structural loads fluctuates with time ( $\sigma = \sigma(t)$ ) as the structure experiences changes in the applied loads. Note that these *applied loads* consist of both the external and inertial forces and moments that are imposed *on* the structure, whereas the *internal structural loads* consist of the forces and moments acting *between* adjoining portions of the structure.

During any given operating interval, an element in the structure may experience many different load intensity levels. Thus, even though the applied load that causes the element to fail may not cause the structure to fail, a subsequent load may still cause the structure to fail before the conclusion of the operating interval. To account for this possibility, the approach uses the *maximum load intensity* for the operating interval to determine whether structural failure has occurred during that interval. To simplify the calculations, the approach also uses this same *maximum load intensity* to determine whether element failure has occurred during the operating interval. The assumptions that justify this later simplification are introduced in this section as Assumptions 4 and 6 (see pp. 15 and 16).

Convention for Measuring the Load Intensity. Because components with different cross-sectional areas will carry different loads even if they have similar locations in the structure, one must consider the loads carried per unit of cross-sectional area. This load per unit area, or *stress*, increases near holes, cutouts, and cracks. Thus, to calculate the local stress at a specific location, the stress analyst often uses a two-step procedure. First, he takes the location of interest and a description of the applied loads and calculates a

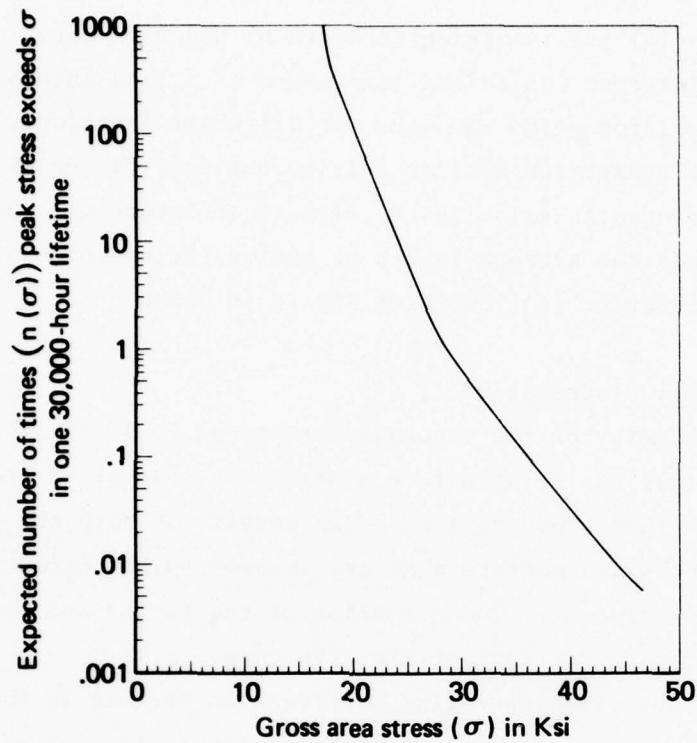


Fig. 2--Load intensity distribution

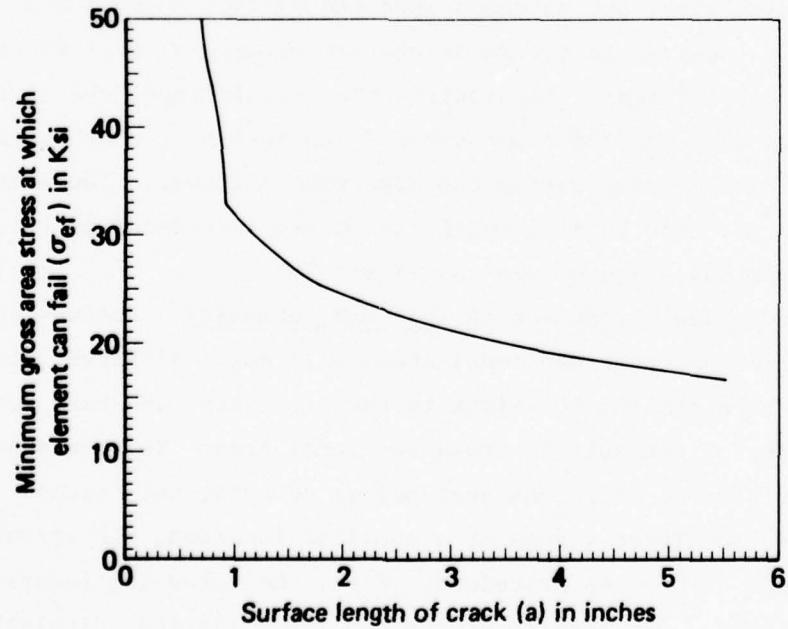
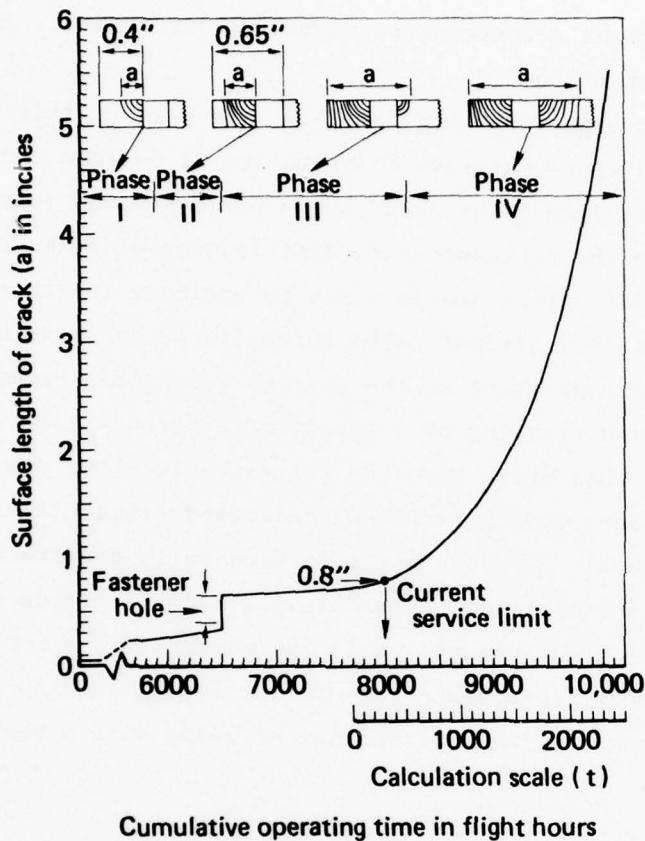


Fig. 3--Remaining strength for the cracked element



Cumulative operating time in flight hours

Fig. 4--Crack propagation history

theoretical stress based on the assumption that holes, cutouts, and cracks do not exist in the structure. By convention, he refers to this theoretical stress as the *gross area stress* because it does not account for holes, cutouts, and cracks that reduce the cross-sectional area available for transmitting loads. Because the structure must transmit the loads around such discontinuities, the actual stresses at such discontinuities usually exceed the gross area stress. Thus, in the second step, he must calculate a correction factor (sometimes referred to as a stress concentration factor) to account for this local redistribution of the load. Even if the stress analyst can accomplish the stress calculation in a single step, he often uses the *gross area stress* as the conventional measure of the *load intensity* that occurs

in the structure as a consequence of the applied loads. This report adopts that convention.

Source for Load Intensity Data. If the user of this methodology does not have a representative distribution of *maximum load intensity* levels (i.e., maximum gross area stress per operating interval), he may have to use the representative distribution of peak-stresses that the fatigue and fracture analyst uses to estimate the rate at which cracks enlarge. For aircraft structures, these analysts usually define the *peak-stress* event as the maximum gross area stress that occurs after each upward crossing of a specified reference level. For transport aircraft, they often base the reference level on the gross area stress that occurs during level unaccelerated flight (i.e., the 1 g gross area stress). Because aircraft frequently deviate from this condition, each operating interval (e.g., flight) yields a set of different peak-stress values. Only the maximum value for a given operating interval represents the *maximum load intensity* required here. Appendix D discusses the implications of using such a peak-stress distribution.

#### Remaining Strength of the Cracked Element

Once the postulated crack reaches a certain length, the remaining strength of the element with the crack decreases as the crack continues to enlarge. Information to represent this phenomenon is easily obtained because the fatigue and fracture analyst must have it to make a *safe-life* calculation. The information may come from laboratory experiments, theoretical calculations, or a combination of both (Refs. 3, 4, and 5).

For a given crack size and identical locations, repeated experiments with different test specimens show relatively little scatter in the observed remaining strength values. A deterministic (rather than a probabilistic) relationship is used to represent this phenomenon. Figure 3 presents the deterministic relationship used in the example application. The surface length ( $a$ ) is used as the measure of crack size, and the gross area stress ( $\sigma$ ) as the measure of load intensity.

In the example, the element is defined to be two panels of the eleven overlapping panels that form the lower surface of the inner wing box. Each is about one foot wide and about one-quarter inch thick. Each has two integral stiffeners; one also has an integral riser. The operator has assumed that both panels might have a fatigue crack propagating from a common fastener location. Furthermore, once one of the panels fails, the other presumably fails. Thus, although Fig. 3 was developed for only a single panel, given the operator specified assumptions, the figure can also be used for the element of interest.

Time Required for Crack Enlargement

The operating time required for the postulated crack to enlarge by a specified amount depends on (1) the material's crack growth characteristics, (2) the operating environment, and (3) the time-varying intensity of the stresses that occur around the crack's leading edge. Although the development of the *crack propagation curve* presented in Fig. 4 requires a fair amount of effort,<sup>1</sup> most airframe manufacturers already develop such curves in applying the *safe-life* philosophy (Ref. 5). The computations required by the probabilistic approach should constitute a relatively small additional burden.

Figure 4 presents a curve that depicts crack size as a function of flight hours. This curve was used to establish the wing's modification schedule. Although the curve starts with an initial crack size having a 0.05 inch corner radius, the probability of element failure is so remote for cracks having a surface length less than 0.8 inch that the probability calculation need not start before the crack is that long. To demonstrate this point, the example calculations will start with an initial crack size having a surface length of 0.7 inch. Figure 4 contains a second time scale with an origin ( $t = 0$ ) at the point where the example calculations start ( $a = 0.7$  inch). Of course, the starting point for the calculation is problem dependent.

It is interesting to observe that the *safe-life* limit for the current wing has been established on the basis of the 8000 flying hours

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<sup>1</sup>See Appendix D for a summary.

required for a 0.05 inch radius corner crack to enlarge to a through-the-thickness crack having a surface length of 0.8 inch.

#### Remaining Strength of the Structure

If the applied load that causes the element of interest to fail also causes the structure to fail, then that applied load has caused the intensity of the internal structural loads to exceed the remaining strength of the structure ( $\sigma_{rs}$ ). Because more than one applied load condition may cause the internal structural loads to have the same load intensity, it is convenient to use the load intensity to measure the remaining strength.

Source for Remaining Strength Data. During the design of many structures, especially airframes, the manufacturer calculates this information. Sometimes he uses a full-scale remaining (residual) strength test for verification. Because repeated tests show relatively little scatter in the results, the methodology uses a deterministic representation for this input.

Data Used in the Example Application. The manufacturer designed the structural members of interest so that if any single panel failed the rest of the structure would not fail provided that (1) the panels adjoining the failed panel contained no significant strength degrading cracks and (2) the applied loads causing panel failure did not cause the load intensity to exceed 42 ksi.

Subsequent to the design of the structure, however, the concern has arisen that during manufacture, an inadvertent action may have damaged two adjoining panels where they overlap at a common fastener hole. In such a case, both members might fail simultaneously. The estimated residual strength ( $\sigma_{rs}$ ) following the postulated dual panel failure is 30 ksi, provided that fatigue cracks have not significantly weakened the panels that are still intact.<sup>1</sup>

#### Treatment of the Uncertainties Implicit in the Inputs

The accuracy of the inputs will vary depending on the particular application. Whenever a significant uncertainty exists, the user

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<sup>1</sup>See Appendix D.

should make appropriate sensitivity analyses by postulating alternative inputs and repeating the calculations according to the procedure described in Sec. III.

UNDERLYING ASSUMPTIONS

To keep the calculation procedure simple and straightforward, a few limitations are imposed so that calculations can be made by hand and thus the basic phenomena influencing the results can be better understood. Each user can add whatever methodological embellishments he finds appropriate for his particular problem. Moreover, minimizing the amount of information required also minimizes opportunities for misinformation to undermine the credibility of the calculated results. Of course, to take results from a simple model and make meaningful extrapolations to real world situations requires a thoughtful analysis of the simplifying assumptions underlying the model.

The first two assumptions help to narrow the scope of the necessary calculations:

*Assumption 1:* Detection of the cracked element does not occur before the crack fails the element.

*Assumption 2:* Once the cracked element fails, a completely effective repair/replacement of that element occurs before the structure begins the next operating interval.

The approximations introduced by the remaining assumptions justify certain simplifications that significantly reduce the complexity of the necessary calculations.

*Assumption 3:* All operating intervals have the same duration.

*Assumption 4:* When each operating interval starts, a single step increase in the crack length provides an adequate representation for the total crack extension that occurs during the operating interval.

Assumption 5: The operations during each operating interval occur independently of the operations during all previous operating intervals.

Assumption 6: If the load intensity exceeds the level required to fail the element more than once during a given operating interval, the highest load intensity level occurs first.

Because the prospective user of this approach must formulate his own analysis strategy to evaluate the implications of these assumptions, Appendix F provides additional discussion of their implications.

### III. DERIVATION OF EQUATIONS

#### THE GENERAL EQUATIONS

For a designated element, postulate that a crack exists at a specified location. According to Assumption 1, detection of the cracked element does not occur before the crack fails the element. Let  $a(t)$  represent the crack's surface length as a function of cumulative operating time ( $t$ ). Define the origin for  $t$  such that  $a(0)$  represents the initial crack length considered by this approach (e.g., Fig. 4). Assumption 3 provides that each operating interval has a constant length (say,  $\Delta t$ ). After  $i$  such intervals, the crack will have a surface length  $a(t_i)$ , where  $t_i \triangleq i \Delta t$ . Because  $a(t)$  varies slowly in an operating interval (except for the interval where element failure occurs), simplify the problem by assuming that the crack size has a constant value of  $a(t_i)$  throughout the  $i$ th interval (Assumption 4).

For the postulated crack site, let  $\sigma(t)$  represent the time-varying intensity of the internal structural loads that the applied loads would cause in an idealized structure without any holes or cracks. Let  $\hat{\sigma}_i$  represent the maximum value of  $\sigma(t)$  that occurs during the  $i$ th operating interval. If  $\hat{\sigma}_i \geq \sigma_{ef}$ , the crack will instantaneously fail the element. If  $\hat{\sigma}_i \geq \sigma_{sf}$ , the structure will immediately collapse. Because both  $\sigma_{ef}$  and  $\sigma_{sf}$  depend on the crack's size (see Figs. 1 and 3), let  $\sigma_{ef} = \sigma_{ef}(a)$  and  $\sigma_{sf} = \sigma_{sf}(a)$  represent these relationships.

The postulated crack can lead to an element failure during the  $j$ th interval only if it has not failed the element during a prior interval. Similarly, because Assumption 2 stipulates that the structure does not start an operating interval with a failed element, the postulated crack can lead to a structural failure during the  $j$ th interval only if it has not led to the element's failure during a prior interval. Thus, for  $x = e$  and  $x = s$ ,

$$\begin{aligned} P_{xf}(t_{j-1}, t_j) &= 1 - P_{nxf}(t_{j-1}, t_j) \\ &= \text{Prob} \left\{ \hat{\sigma}_i < \sigma_{ef}(a(t_i)); i=1,2,\dots,j-1 \right\} \text{Prob} \left\{ \hat{\sigma}_j \geq \sigma_{xf}(a(t_j)) \right\} \end{aligned} \quad (1)$$

denotes the probability that the postulated crack leads to a type  $x$  failure during the  $j$ th interval, where  $x = e$  for an element failure and  $x = s$  for a structural failure. Justification for the product contained in the right-hand side of Eq. (1) comes from the fact that  $\hat{\sigma}_j$  must not depend on  $\hat{\sigma}_i$  ( $i = 1, 2, \dots, j-1$ ) because Assumption 5 stipulates that the operating conditions during the  $j$ th interval occur independently of the operating conditions during prior intervals; note that the operating conditions govern  $\sigma(t)$  and hence  $\hat{\sigma}_j$ .

For  $k$  consecutive operating intervals,  $k$  different outcomes may occur for failure type  $x$ . Thus

$$P_{nxf}(0, t_k) = 1 - \sum_{j=1}^k P_{xf}(t_{j-1}, t_j); (x = e, s) \quad (2)$$

denotes the probability that during the time  $(0, t_k)$  the postulated crack condition will not lead to a type  $x$  failure. When  $x = e$ , and as long as  $\text{Prob}\{\hat{\sigma}_j \geq \sigma_{ef}(a(t_j))\} \ll 1$ , the approximation

$$P_{nxf}(0, t_k) = \prod_{j=1}^k \left[ 1 - \text{Prob}\left\{\hat{\sigma}_j \geq \sigma_{ef}(a(t_j))\right\} \right] \quad (3)$$

provides a useful replacement for Eqs. (1) and (2).

For convenience, define

$$r_{xf}(t_i) \triangleq n(\sigma_{xf}(a(t_i)))/t_d; (x = e, s), \quad (4)$$

where  $n(\sigma_{xf})$  represents the expected number of times that the load intensity would exceed  $\sigma_{xf}$  during one design lifetime of operation (e.g., see Fig. 2). The design lifetime ( $t_d$ ) must have the same units as  $\Delta t$ . For the  $j$ th operating interval,  $r_{xf}(t_j) \Delta t$  represents the expected number of times that the load intensity would exceed  $\sigma_{xf}(a(t_j))$  during one operating interval. Because all operating intervals have the same fixed duration (Assumption 3), these intervals must be

uniformly distributed over the cumulative operating time. Thus, given Assumption 6, the uniform distribution and independence properties that stem from Assumptions 3 and 5 and the restriction that  $r_{xf}(t_j) \Delta t \ll 1$ , the relation

$$\text{Prob} \left\{ \hat{\sigma}_j \geq \sigma_{xf}(a(t_j)) \right\} = r_{xf}(t_j) \Delta t \quad (5)$$

provides a reasonable approximation for our purposes.

If  $\hat{\sigma}_j \geq \sigma_{ef}(a(t_j))$ , the element of interest will fail; the structure will also fail if  $\hat{\sigma}_j \geq \sigma_{rs}$  (see the dashed line in Fig. 1). Define  $\tilde{t}$  as the time when  $\sigma_{ef}(a(\tilde{t})) = \sigma_{rs}$  (Fig. 1). Prior to time  $\tilde{t}$ ,  $\sigma_{ef}(a(t_j)) > \sigma_{rs}$ , and thus  $\sigma_{sf}(a(t_j)) = \sigma_{ef}(a(t_j))$ . After time  $\tilde{t}$ ,  $\sigma_{ef}(a(t_j)) < \sigma_{rs}$ , and thus  $\sigma_{sf}(a(t_j)) = \sigma_{rs}$ . Incorporate these results in Eq. (4) and recognize that  $n(\sigma_{xf})$  decreases as  $\sigma_{xf}$  increases, whence

$$r_{sf}(t_j) = \min \left\{ r_{ef}(t_j), n(\sigma_{rs})/t_d \right\}. \quad (6)$$

Thus, from Eqs. (3), (4), and (5),

$$P_{nef}(0, t_k) = \prod_{j=1}^k \left\{ 1 - r_{ef}(t_j) \Delta t \right\} \quad (7a)$$

and from Eqs. (1), (2), and (5),

$$P_{nsf}(0, t_k) = 1 - \sum_{j=1}^k P_{nef}(0, t_{j-1}) r_{sf}(t_j) \Delta t. \quad (7b)$$

Equations (6) and (7) present the general result. Appendix G gives an alternative formulation that uses integrals. One might use that form along with simple expressions for  $n(\sigma)$ ,  $\sigma_{ef}(a)$ , and  $a(t)$  to obtain an approximate analytical solution.

AN APPROXIMATE PROCEDURE FOR HAND CALCULATIONS

The values for  $r_{ef}$  and  $r_{sf}$  vary in a sufficiently slow manner that one can make a useful approximate calculation by hand if he assumes that the values for  $r_{ef}$  and  $r_{sf}$  remain constant over a sequence of consecutive operating intervals. Let such a sequence define a calculation interval; assign a value to the index  $\alpha$  to represent each calculation interval ( $\alpha = 1, 2, 3, \dots$ ). Assume that each calculation interval contains an even number of operating intervals; let  $2k_\alpha$  represent that number. Thus, the midpoints for successive calculation intervals (e.g.,  $\bar{t}_\alpha$ ) are given by

$$\bar{t}_\alpha = \bar{t}_{\alpha-1} + (k_{\alpha-1} + k_\alpha) \Delta t , \quad (8)$$

where  $k_0 \triangleq 0$  and  $\bar{t}_0 \triangleq 0$ .

Let the last operating interval that occurs prior to time  $\bar{t}_\alpha$  have an index  $L_\alpha$ . From Eq. (8),

$$L_\alpha = k_\alpha + \sum_{\beta=0}^{\alpha-1} 2k_\beta .$$

Assume that  $r_{ef}$  and  $r_{sf}$  have constant values equal to  $r_{ef}(\bar{t}_\alpha)$  and  $r_{sf}(\bar{t}_\alpha)$  during calculation interval  $\alpha$ . Thus, for the last half of interval  $\alpha - 1$ ,

$$\prod_{j=L_{\alpha-1}+1}^{L_{\alpha-1}+k_{\alpha-1}} \left\{ 1 - r_{ef}(t_j) \Delta t \right\} \approx \left\{ 1 - r_{ef}(\bar{t}_{\alpha-1}) \Delta t \right\}^{k_{\alpha-1}} , \quad (9a)$$

and

$$\sum_{j=L_{\alpha-1}+1}^{L_{\alpha-1}+k_{\alpha-1}} \frac{P_{nef}(0, t_j)}{1 - r_{ef}(t_j) \Delta t} r_{sf}(t_j) \Delta t = k_{\alpha-1} \frac{P_{nef}(0, \bar{t}_{\alpha-1})}{1 - r_{ef}(\bar{t}_{\alpha-1}) \Delta t} r_{sf}(\bar{t}_{\alpha-1}) \Delta t .$$

(9b)

Similarly, for the first half of interval  $\alpha$

$$\prod_{j=L_\alpha-k_\alpha+1}^{L_\alpha} \left\{ 1 - r_{ef}(t_j) \Delta t \right\} = \left\{ 1 - r_{ef}(\bar{t}_\alpha) \Delta t \right\}^{k_\alpha} \quad (9c)$$

and

$$\sum_{j=L_\alpha-k_\alpha+1}^{L_\alpha} \frac{p_{nef}(0, t_j)}{1 - r_{ef}(t_j) \Delta t} r_{sf}(t_j) \Delta t = k_\alpha \frac{p_{nef}(0, \bar{t}_\alpha)}{1 - r_{ef}(\bar{t}_\alpha) \Delta t} r_{sf}(\bar{t}_\alpha) \Delta t. \quad (9d)$$

Thus, from Eqs. (7) and (9),

$$p_{nef}(0, \bar{t}_\alpha) = p_{nef}(0, \bar{t}_{\alpha-1}) \left\{ 1 - r_{ef}(\bar{t}_{\alpha-1}) \Delta t \right\}^{k_{\alpha-1}} \left\{ 1 - r_{ef}(\bar{t}_\alpha) \Delta t \right\}^{k_\alpha}, \quad (10a)$$

$$\begin{aligned} p_{nsf}(0, \bar{t}_\alpha) &= p_{nsf}(0, \bar{t}_{\alpha-1}) - k_{\alpha-1} \frac{p_{nef}(0, \bar{t}_{\alpha-1})}{1 - r_{ef}(\bar{t}_{\alpha-1}) \Delta t} r_{sf}(\bar{t}_{\alpha-1}) \Delta t \\ &\quad - k_\alpha \frac{p_{nef}(0, \bar{t}_\alpha)}{1 - r_{ef}(\bar{t}_\alpha) \Delta t} r_{sf}(\bar{t}_\alpha) \Delta t. \end{aligned} \quad (10b)$$

A calculation should start (i.e.,  $t = 0$ ) with a sufficiently small crack size that  $p_{nef}(-\infty, 0) \approx 1.0$ , and  $p_{nsf}(-\infty, 0) \approx 1.0$ . In such a case, one can prudently commence the calculation with the initial conditions  $p_{nef}(0, \bar{t}_0) = 1.0$  and  $p_{nsf}(0, \bar{t}_0) = 1.0$ . With such initial conditions, Eqs. (4), (6), and (10) prescribe all of the calculations made in Sec. IV.

IV. EXAMPLE CALCULATION

This example illustrates the application of our method to an element that forms part of an airframe's exterior surface. The operator of this airframe has postulated a crack condition that he has used to establish limits for the flying hours that he will allow this airframe to accumulate before the manufacturer incorporates a major structural modification that includes replacing the element of interest.

The operator has assumed that the postulated crack condition could result from a faulty manufacturing operation at a fastener hole located along a particular joint where two adjoining panels overlap. The operator has reasoned that, because the holes for each fastener were drilled through both panels in a single manufacturing operation, both panels could have sustained damage from a faulty drilling or reaming operation. Moreover, if the damage was sufficiently severe, the fastener might not have achieved the kind of interference fit that would otherwise prolong fatigue life and retard crack propagation. Furthermore, if the panels sustained enough damage, the fasteners may not have firmly clamped the panels together. Without such clamp-up, friction forces might not develop between the panels; in that case, the fasteners (assumed not to have failed, of course) would have to transfer all of the loads between the panels. Such a fastener load transfer would accelerate the early phase of the crack's development.

To represent the effects of such a rogue condition, the operator specified that the manufacturer should assume that during fabrication, a crack with a 0.05 inch surface length existed at the postulated location (see the Phase I sketch in Fig. 4). To establish the maximum acceptable size to which the operator would allow this hypothetical crack to grow, he specified that the manufacturer should use the limit load as the failing load condition. The limit load gross area stress of 42 Ksi yielded a maximum acceptable crack length of about 0.8 inch. The operator had reasoned that if cracks developed from a common fastener located in two adjoining panels, and if one of the cracks reached a length of about 0.8 inch, then a limit load condition would

probably fail both of the panels as well as cause a catastrophic collapse of the structure. Moreover, an overlapping panel could obscure a crack less than 1.6 inches. It might also obscure signs of fuel leakage that might otherwise signify the presence of a crack.

Using the representative operational use from a recent year, and the configuration of the aircraft at that time, the manufacturer calculated that it would take 8000 flight hours for the 0.05 inch corner crack to enlarge to a 0.8 inch surface length (see Fig. 4). The operator has subsequently stipulated that no aircraft will accumulate more than 8000 flight hours before a major modification replaces this portion of the structure.

#### THE ISSUE

If the operator postpones this modification and uses this aircraft beyond the current 8000-hour limit, what chances does he take that a catastrophic structural failure will occur? Consider the following situation: (1) Suppose the postulated crack condition develops in the two adjoining panels. (2) Also, suppose that detection of these cracks does not occur before they fail both panels. What is the probability that such a dual panel failure will not lead to the catastrophic collapse of the structure?

For this situation, the subsequent calculation shows that there is a 97.4 percent chance that the structure will not collapse.

#### THE CALCULATION

The inputs consist of Figs. 2 through 4, an estimated residual strength of  $\sigma_{rs} = 30$  Kpsi for the postulated dual panel failure (see Appendix D), and an assumed flight duration  $\Delta t = 5$  hours. The design lifetime on which the values for  $n$  are based (Fig. 2) is  $t_d = 30,000$  hours.

To record the input information in Table 1, use the following procedure for each calculation interval:

1. Select the number of flights ( $2k_\alpha$ ) for the current calculation interval ( $\alpha$ ) and use Eq. (8) to calculate the midpoint ( $\bar{t}_\alpha$ )

Table 1

## EXAMPLE CALCULATION FOR A DUAL PANEL FAILURE

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Calculation Interval $\alpha$	$\bar{t}_\alpha$ Eq. (8)	$k_\alpha$ (2)	$a(\bar{t}_\alpha)$ Fig. 4	$\sigma_{ef}(a)$ Fig. 3	$n(\sigma_{ef})$ Fig. 2	Preliminary Calculations [Eqs. (4) and (6)]		Probability of No Element Failure [Eq. (10a)]		Probability of No Structural Failure [Eq. (10b)]	
						$r_{sf}(\bar{t}_\alpha) \Delta t$ $= \min(\textcircled{7},$ $\textcircled{6} \frac{\Delta t}{t_d} n(\sigma_{rs}) \frac{\Delta t}{t_d})$	$\textcircled{9} =$ $1 - \textcircled{7}$	$\textcircled{10} =$ $\textcircled{9} \textcircled{2}$	$P_{nef}(0, \bar{t}_\alpha) =$ $\textcircled{11}_{\alpha-1} \textcircled{10}_{\alpha-1}$ $\times \textcircled{10}_\alpha$	$\textcircled{12} =$ $\textcircled{2} \textcircled{8} \textcircled{11}$ $\textcircled{9}$	$P_{nsf}(0, \bar{t}_\alpha) =$ $\textcircled{12} -$ $\textcircled{12}_\alpha$
1	20	100	.74	46.5	.00575	$9.58 \times 10^{-7}$	$9.58 \times 10^{-7}$	1.0000	1.0000	.0000	1.0000
2	20	300	.79	44.0	.00960	$1.60 \times 10^{-6}$	$1.60 \times 10^{-6}$	1.0000	1.0000	.0000	1.0000
3	20	500	.89	36.0	.100	$1.67 \times 10^{-5}$	$1.67 \times 10^{-5}$	1.0000	.9997	.0003	.9997
4	20	700	1.04	31.5	.380	$6.33 \times 10^{-5}$	$6.33 \times 10^{-5}$	.9999	.9981	.0013	.9981
5	20	900	1.22	29.8	.620	$1.03 \times 10^{-4}$	$9.75 \times 10^{-5}$	.9999	.9947	.0019	.9949
6	20	1100	1.45	27.5	1.50	$2.50 \times 10^{-4}$	$9.75 \times 10^{-5}$	.9998	.9950	.0019	.9911
7	20	1300	1.71	25.8	3.90	$6.50 \times 10^{-4}$	$9.75 \times 10^{-5}$	.9994	.9871	.0019	.9873
8	10	1450	1.98	24.9	6.40	$1.07 \times 10^{-3}$	$9.75 \times 10^{-5}$	.9989	.9894	.0009	.9845
9	10	1550	2.18	23.8	10.3	$1.72 \times 10^{-3}$	$9.75 \times 10^{-5}$	.9983	.9830	.0009	.9827
10	10	1650	2.41	23.2	17.7	$2.95 \times 10^{-3}$	$9.75 \times 10^{-5}$	.9971	.9709	.0009	.9809
11	10	1750	2.67	22.3	29.0	$4.83 \times 10^{-3}$	$9.75 \times 10^{-5}$	.9952	.9527	.0008	.9792
12	10	1850	2.97	21.4	47.0	$7.83 \times 10^{-3}$	$9.75 \times 10^{-5}$	.9922	.9244	.0007	.9777
13	10	1950	3.32	20.5	80.0	$1.33 \times 10^{-2}$	$9.75 \times 10^{-5}$	.9867	.8744	.0006	.9764
14	10	2050	3.73	19.6	137	$2.28 \times 10^{-2}$	$9.75 \times 10^{-5}$	.9772	.7938	.0004	.9754
15	5	2125	4.08	19.0	197	$3.28 \times 10^{-2}$	$9.75 \times 10^{-5}$	.9672	.8463	.0001	.9749
16	5	2175	4.33	18.6	250	$4.17 \times 10^{-2}$	$9.75 \times 10^{-5}$	.9583	.8083	.0001	.9747
17	5	2225	4.61	18.1	375	$6.25 \times 10^{-2}$	$9.75 \times 10^{-5}$	.9375	.7242	.0001	.9745
18	5	2275	4.92	17.7	620	$1.03 \times 10^{-1}$	$9.75 \times 10^{-5}$	.897	.580	.0454	.9744
19	5	2325	5.25	17.3	880	$1.47 \times 10^{-1}$	$9.75 \times 10^{-5}$	.853	.452	.0119	.9744
20	5	2375	5.60	16.8	1400	$2.33 \times 10^{-1}$	$9.75 \times 10^{-5}$	.767	.265	.0014	.9744

for this interval; record the values for  $\alpha$ ,  $k_\alpha$ , and  $\bar{t}_\alpha$  in columns 1 through 3.

2. Use  $t = \bar{t}_\alpha$  and enter the lower scale in Fig. 4 to obtain the crack length ( $a(\bar{t}_\alpha)$ ); record this value in column 4.
3. Take this crack length ( $a(\bar{t}_\alpha)$ ) and enter Fig. 3 to obtain the gross area stress at which element failure would occur ( $\sigma_{ef}(a)$ ); record this value in column 5.
4. Take this gross area stress ( $\sigma_{ef}(a)$ ) and enter Fig. 2 to obtain the number of times that the load intensity would exceed this level in one 30,000 hour lifetime of operation ( $n(\sigma_{ef})$ ); record this value in column 6.

To make the necessary preliminary calculations

1. Use Eq. (4) to calculate  $r_{ef}(\bar{t}_\alpha) \Delta t$ ; record this value in column 7.
2. Use Eq. (6) to calculate  $r_{sf}(\bar{t}_\alpha) \Delta t$ ; record this value in column 8.

To complete the probability calculations, assume that  $P_{nef}(0, \bar{t}_0) = 1.0000$  and  $P_{nsf}(0, \bar{t}_0) = 1.0000$  and carry out the calculations prescribed by Eqs. (10a) and (10b). Table 1 contains the pertinent values in columns 9 through 13. The values in columns 10 and 12 support the assumed initial conditions for the four significant figures displayed; the first two intervals offer no contribution to the results. Moreover, the monotonic behavior of the inputs assures us that prior intervals would not contribute either.

To minimize the error caused by the assumption that  $r_{ef}$  remains constant throughout a given calculation interval, the size of the calculation interval decreases as  $r_{ef}$  increases.

#### THE RESULTS

Columns 11 and 13 contain the desired results. Table 2 summarizes these results for selected times and corresponding crack lengths. Note that the first column in Table 2 presents the cumulative flight

Table 2  
SUMMARY OF RESULTS

Cumulative Flight Time (hr)		Probability that the Dual Panel Crack Condition Does Not Lead to--		
From a 0.05 Inch Crack	From the Start of the Calcula- tion (t)	Crack Length in Inches (a)	Element Failure by Time t $P_{nef}(0,t)$	Structural Failure by Time t $P_{nsf}(0,t)$
7900	300	0.8	1.000	1.000
8500	900	1.2	0.995	0.995
8800	1200	1.6	0.98	0.989
9050	1450	2.0	0.95	0.985
9600	2000	3.5	0.50	0.976
9875	2275	4.9	0.05	0.974
9925	2325	5.3	0.01	0.974

time measured relative to the time when the postulated crack reaches a 0.05 inch length. When a given crack would reach such a length depends on the conditions that cause the crack to start. However, because the operator of this aircraft believes that such a crack might conceivably start in a new aircraft, he has adopted the cumulative flight hours in column 1 and the crack length in column 3 to establish the maximum number of hours that he will allow this aircraft to accumulate. To set this service limit, the operator in effect judged that the probability of the crack's leading to an element failure would increase rapidly soon after the crack had reached a limit load critical length of about 0.8 inch. Indeed, the results for  $P_{nef}$  show a rapid decline that develops about a thousand hours after the crack reaches a length of 0.8 inch, thus confirming this aspect of the operator's concern.

The operator, however, had an additional concern that simultaneous failure of the two panels would probably lead to immediate failure of the structure. The results for  $P_{nsf}$  in Table 2 show a 97.4 percent chance that the structure would not fail provided that the previously stipulated assumptions hold (see Sec. II). For example, detection of

the crack does not occur before the element fails, and detection of an element failure occurs prior to the next flight. Table 2 shows a 98 percent chance that an element failure should not occur before the postulated crack reaches a length of 1.6 inches. Once it reaches that length, it becomes visible from outside the aircraft and detection because of fuel leakage becomes increasingly more probable. Assuming detection and repair at that point, the operator suffers at most a 1.1 percent chance of losing the aircraft prior to that time. Note that Table 2 shows a 3.5 inch expected mean value for the crack length at the time element failure occurs.

Even if the postulated crack reaches a 5 inch (or even longer) length without being detected, the structure has a 97.4 percent chance of not failing. Moreover, Table 2 shows that most of the 2.6 percent chance that it will fail occurs while the crack has a length between 1.2 and 3.5 inches, and over half of this threat occurs after the crack has exceeded a length of 1.6 inches. Thus, based on these calculations, the operator no longer needs to assume that a crack will necessarily lead to a panel failure before it reaches a length of 1.6 inches. Furthermore, even if a crack does progress undetected and eventually leads to a dual panel failure, there is only a small chance that the structure will fail.

#### SENSITIVITY OF THE RESULTS

If only a single panel fails, the structure has a residual strength of  $\sigma_{rs} = 42$  ksi (rather than 30 ksi). In this case, the structure has a 99.9 percent chance of not failing before the crack reaches a length of 1.6 inches (see Appendix H). Even if the postulated crack is assumed to develop without being detected, the structure still has a 99.9 percent chance of not failing if only a single panel fails.

Assumption 1 has an obvious influence on this example (see Sec. II). Once the postulated crack reaches a length of 1.6 inches, the overlapping panel can no longer block an inspector's view of the crack. Moreover, fuel leakage from a wing fuel tank becomes increasingly more likely as the crack enlarges beyond that point. Although the dual panel crack condition might be detected during an internal inspection of the wing

structure, the detection probability increases significantly once the crack can be detected by external inspection. Recall that Table 1 shows that over half of the 2.6 percent chance that the structure will fail occurs after the crack has reached a length of 1.6 inches. Because it typically takes 800 hours for the crack to advance from 1.6 to 3.5 inches, frequent external inspections for cracks having lengths in that range could help to reduce the chances of a catastrophic structural failure.

Other factors that may influence this example include the assumptions underlying the curves in Figs. 2 and 4. Those curves represent an aircraft configuration and operational use that causes higher stresses than the current operation of the currently configured aircraft. Thus, for the current aircraft, the postulated crack would enlarge at a significantly slower rate than indicated in Fig. 4. Moreover, the current aircraft will less frequently exceed the gross area stress levels at which element failure and structural failure would occur. One might expect that these factors should reduce the chances of a catastrophic structural failure. However, to generate a meaningful sensitivity analysis, the manufacturer would probably have to first provide current curves for both Figs. 2 and 4.<sup>1</sup>

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<sup>1</sup>See Appendix I for additional suggestions.

V. OBSERVATIONS AND RECOMMENDATIONS

For the rarely occurring postulated crack condition examined here, it is 38 times more probable that a dual panel failure will occur *without* structural failure than *with* structural failure. Thus, subject to the stipulated assumptions, one would expect the postulated crack condition to lead to 38 dual panel failures that do *not* cause a structural failure for every such dual panel failure that does cause a structural failure. However, if the postulated crack condition only extends to a single panel, then one would expect 1000 single panel failures that do not cause a structural failure for every such single panel failure that does. The historical experience of similarly constructed aircraft as well as the cracks detected in the fatigue test article for this aircraft suggest that single panel failures should occur much more frequently than dual panel failures. Therefore, before a catastrophic structural failure occurs, one can expect a large number of single or even dual panel failures to occur first.

Moreover, recall that the example used a worst case element location in the sense that the location had the shortest crack propagation time and therefore governed the current service limit. Other wing panels on this aircraft experience lower stresses and thus have larger crack propagation times and higher probabilities that the same kind of crack will not lead to a structural failure. Thus, if the postulated crack condition existed at a random location, rather than a worst location, one would expect numbers much larger than the 38 and 1000.

This damage tolerance exhibited by the aircraft examined in the example does not necessarily extend to all aircraft structures. For example, whereas the aircraft in the example has eleven independent panels comprising the primary load-carrying structure of the wing's lower surface, some aircraft have only a single panel supported by perhaps a dozen stiffeners and in some cases the stiffeners can only carry a small fraction of the lower surface loads. Consequently, for such a structure a single panel failure necessarily causes structural failure. Other aircraft have three or four panels joined by internal

splicing stiffeners and spar caps that have a significant load-carrying capability. If the failure of an internal component goes undetected, the crack development and enlargement processes in the adjoining panel will accelerate, and a dual component failure becomes a matter of serious concern. Because aircraft structures are not usually designed to tolerate a dual component failure, many of them will fail if two adjoining components fail.

The aircraft considered in the example, therefore, has two significant virtues. First, because it has no internal (hidden) primary load-carrying components (except for doublers) that comprise the wing's lower surface, a single component failure is not likely to go undetected and thus should not contribute to a dual component failure mode. Second, even if two adjoining components in the critical area fail, the structure will not fail with probability 0.974. Thus, even a dual component failure will probably not cause a structural failure. Some aircraft, however, have components susceptible to the dual component failure mode, and they have a low chance of surviving such a failure. This wide range of damage tolerance capabilities exists because the kind of probabilities calculated here have not been calculated as an integral part of the structural design process. We recommend, therefore, that operators require manufacturers to use a probabilistic approach, such as the one presented here, to help evaluate the need for major structural modification programs. This would assure that the modification fits into a broader strategy to manage the threat posed by seldom occurring fatigue cracks that can appear in any aircraft the operator uses.

The approach proposed here provides a capability to help assess the risk incurred when a crack develops sooner than expected, or when aircraft arrive at a service limit in an undamaged state and continue in operation beyond that limit. Currently, some approaches to establishing service limits attempt to assess when a crack with a limit load critical crack length might develop in the structure. To establish a reasonably safe limit, some operators require the manufacturer to incorporate conservative assumptions in the calculation process. Nonetheless, this cannot preclude the possibility that a crack might reach

a limit load critical length sooner than expected. Furthermore, as a consequence of the necessarily conservative assumptions incorporated in the assessment process, an aircraft might arrive at the specified service limit without exhibiting any significant evidence of service-limiting fatigue cracks in the structure. In such an event, the operator might decide to use the aircraft beyond the original limit.

The calculation of a safe service limit based on an assumed initial flaw size provides a reasonable and useful procedure for controlling the design stress levels and materials used in future aircraft. It also provides a preliminary estimate for a service limit. Our probabilistic approach provides a procedure for assessing the structure's ability to tolerate element failures resulting from undetected fatigue crack propagation. Thus, we recommend that operators and manufacturers augment their service limit assessment procedures with a probabilistic approach such as the one presented here to provide a superior approach for managing the risk presented by unexpected and undetected fatigue cracks.

Application of this approach during the design phase could help to assure uniform damage tolerance capabilities for new structures. Structural maintenance personnel could use the same kind of information to accomplish a more cost-effective allocation of resources used to maintain and improve the damage tolerance of operational structures. People who either operate, or certify the operation of, structures could use this damage tolerance information to assess the relative risk of operating different structures after it is first suspected that an isolated element may contain a service-induced fatigue crack. Thus, we recommend that manufacturers adopt a probabilistic approach such as the one presented here to help inform management about a structure's damage tolerance capability *throughout the structure's life cycle--from early design through final retirement.*

Appendix A

CATASTROPHIC FAILURE OF AIRCRAFT STRUCTURES

Fatigue cracks that cause airframe structures to fail usually start where the geometry of an element causes a local concentration of stresses. A further concentration of stresses sometimes occurs as a result of a faulty manufacturing or maintenance operation. The greatest stress concentration frequently occurs at faulty fastener holes, because even for a properly prepared hole under elastic conditions the maximum stress can equal three times the stress at a location one or two hole diameters away. Thus, a fastener hole with a manufacturing or maintenance irregularity constitutes a prime source for a fatigue crack, especially if the hole occurs in a highly stressed element.

Although fatigue cracks can eventually develop in fastener holes that do not have any manufacturing irregularities, this report focuses mainly on the early cracks that can develop at fastener holes having irregularities. Examples of the relevant irregularities include burrs, radial tears, rifle marks, gouges, and improperly installed fasteners. The method presented in this report can assess the probability that a structural failure would eventually occur if an unexpected fatigue crack should enlarge without being detected.

Regardless of the source, during a crack's early development phase its size increases slowly and because of its small size, it does not noticeably decrease the overall strength of the structure. In many cases, however, fatigue cracks eventually reach a size where they can appreciably influence strength. Sometimes catastrophic structural failure results.

RECENT EXAMPLES

Boeing 707 Near Lukasa, Zambia

On May 14, 1977, the right side of the horizontal tail of an all-cargo configured 707-321 collapsed following an in-flight element

failure that occurred while approaching the runway at Lukasa. The aircraft, operated by Dan Air, had accumulated 47,000 flight hours. Following the collapse, it entered a steep dive and crashed. None of the five occupants survived (Refs. 6, 7, and 8).

Lockheed C-141A Near Mildenhall, England

On August 28, 1976, severe atmospheric turbulence apparently caused the catastrophic structural failure of a C-141A wing. Manufactured by Lockheed and operated by the USAF Military Airlift Command, the aircraft had accumulated 15,372 flight hours. The wing failure occurred at about 10,000 feet as the aircraft approached Mildenhall Air Force Base. None of the 18 occupants survived (Ref. 9). Investigators examined pieces of the failed wing that had not been consumed by fire but could find no evidence of fatigue cracks that would have weakened the wing. The failure may have occurred when turbulence imposed an aerodynamic load on the wing that would have even exceeded the strength capability of a new wing.

This example illustrates that aircraft structures do, on occasion, experience an extraordinarily high applied load.

Lockheed L-382 Near Springfield, Illinois

On May 23, 1974, an all-cargo configured version of the USAF/Lockheed C-130 operated by Saturn Airways had the outboard section of the left wing collapse after an element failure. The resulting imbalance in the aerodynamic loads caused the aircraft to enter a violent roll and yaw that overloaded the outboard section of the right wing, thus causing it to collapse also. The aircraft then descended at a steep angle from 21,000 feet and crashed; none of the four occupants survived. The aircraft, which had had its fuselage lengthened by 80 inches in 1972, had accumulated 18,837 flight hours (Refs. 10 and 11).

POTENTIAL ROLES FOR THE CALCULATION PROCEDURE

The calculation procedure proposed by this report might help to avert some failures like those just discussed. For example, the failure of an obscured or partially visible spar cap apparently led

to both the 707 and the L-382 failures (Refs. 6, 7, 8, 10, and 11). To apply this probabilistic procedure to an obscured component, one would have to accept the obscured component's failure as a given and postulate the existence of the fatigue crack in an adjoining unobscured component such as the surface panel that obscures the spar cap. By calculating the probability that such a dual component failure would not lead to a structural failure, one could assess the structure's vulnerability to such a failure mode. This could help indicate the relative importance of establishing special inspection and modification actions for the most critical areas of the structure.

Hindsight now tends to suggest that the spar cap areas where the cited failures occurred are indeed critical; moreover, these areas may not have received as much attention as they deserved (Refs. 6, 10, and 11). It would be interesting to find out whether this calculation procedure could have helped to provide some degree of foresight.

Because the C-141A and KC-135 also have obscured spar caps, it might be interesting to apply this procedure to those aircraft as well.

Appendix B

THE PROBABILISTIC APPROACH AND SHORTCOMINGS  
OF CURRENT APPROACHES

Both the *fail-safe* and the *safe-life* philosophies have desirable design features. The *fail-safe* forces the designer to use multiple independent elements to transmit loads through a structure; the *safe-life* forces him to use materials and design stress levels that should preclude the possibility that the specified initial crack might reach the critical length during the aircraft's specified operating life. The shortcomings of both philosophies, however, have stimulated debate.

THE FAIL-SAFE PHILOSOPHY

The success of the *fail-safe* philosophy hinges on the assumption that no cracks exist in the structure adjoining a failed element. Of course, the validity of such an assumption cannot be assured in all situations. For example, consider the L-382 incident described in Appendix A. Two elements adjacent to the first element that failed contained cracks in the immediate neighborhood of the initial failure site. Because of these cracks, the elements could not stop the progression of the failure.<sup>1</sup>

The National Transportation Safety Board (NTSB) report does not say whether these cracks developed before or after the initial element failure (Ref. 11). If the initial element failure occurred first, then the failure to promptly discover that condition (e.g., within a certain number of flights) provided the time for the additional loads (that the failed element had transferred to the adjoining elements) to cause the accelerated development of the fatigue cracks found in those elements.

If the cracks developed before initial element failure occurred, however, the accumulated operational use of that portion of the

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<sup>1</sup>See Appendix A for how the proposed procedure might have been applied to this situation.

structure had exceeded the amount of use necessary to cause the indicated general state of cracking. In either case, proponents of the safe-life philosophy can claim that the accident might not have occurred if the manufacturer and the operator had limited aircraft operation to a calculated safe-life interval, because the initial element failure should not have occurred during such an interval.

Furthermore, the safe-life advocates can argue that the fail-safe philosophy has another Achilles' heel in the applied load used to design the structure to meet the fail-safe criterion. If an applied load exceeds this fail-safe design load, and if that load fails the element of interest, then a structural failure will occur. No matter how high a design load one selects, an applied load might exceed it.

#### THE SAFE-LIFE PHILOSOPHY

The success of the safe-life philosophy hinges on the assumption that the specified length of the initial crack adequately accounts for all design/manufacturing/maintenance errors that might occur. Moreover, for any initial crack condition that the safe-life advocates postulate, the fail-safe advocates can postulate the occurrence of a more severe condition. To soften that argument, the safe-life advocates can postulate an extremely severe, and rarely occurring, initial crack condition (sometimes referred to as a *rogue flaw*). Nonetheless, no matter how severe a *rogue flaw* assumption one makes, someone else can conjure up an even more disastrous situation that might occur. For example, in addition to the assumed crack length, the *rogue flaw* condition often presumes a very extreme situation, for example: (1) least inspectable crack location; (2) highest crack growth rate location; (3) highest crack growth rate environment; (4) no crack growth retardation benefit from interference fit fasteners; and (5) no load transfer benefits from the friction between overlapping structural components. Nonetheless, even worse conditions might occur, albeit with ever-decreasing probabilities of occurrence; for example, the strength of the material may fall below specifications.

To provide reasonable assurance that cracks will not develop to a size where they would threaten the structural integrity of new designs,

the U.S. Air Force has established design requirements based on the *rogue flaw* concept and a *safe-life* calculation (Ref. 1), together with considerations about fail safety. The Air Force has also used this approach to estimate how long it can safely use existing airframes before modification or replacement of components (Ref. 2). The United Kingdom's Airworthiness Requirements Board (ARB) has also considered using the approach to help establish mandatory retirement dates for older aircraft (Refs. 12 and 13). The UK ARB concern stems from incidents where the wings came off an F-27 and an HS-748; both aircraft were thought to have "fail-safe" designs. Moreover, the pressure cabin failures of a Herald and a Vanguard, as well as the 707 incident in Zambia,<sup>1</sup> have increased the ARB's concern.

The current Air Force approach requires a fixed extreme value specification for two probabilistic phenomena: (1) the occurrence of a rogue flaw and (2) the occurrence of the applied load that causes failure. The latter may come from an unusually severe maneuver or extreme atmospheric turbulence, or a combination of both. Thus, the success of this kind of *extreme value approach* depends on whether the postulated applied load represents a sufficiently extreme and rarely occurring condition (sometimes referred to as the *rogue load condition*). But no matter how severe the set of rogue flaw and rogue load assumptions one makes, an even more serious pair of situations might occur that the *safe-life* approach does not cover. Although an approach that explicitly deals with the likelihood of such situations occurring may appear superior to an *extreme value* approach, the validity of such an approach depends on the validity of the inputs.

#### A PROBABILISTIC APPROACH

Because dependable probabilistic information seldom if ever exists for the likelihood of a rogue flaw occurrence, the probabilistic approach proposed here circumvents the rogue flaw issue and uses a probabilistic representation only for the applied load.

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<sup>1</sup> See Appendix A for how the proposed procedure might have been applied to this situation.

The *safe-life* value calculated by an extreme value approach depends strongly on the extent of the postulated rogue flaw condition. And, unfortunately, the limited data from crack distributions observed in service aircraft frequently cannot support attempts to assess the likelihood that a particular rogue flaw condition will occur in a specific element of a given structure. Thus, to postulate an initial equivalent crack length to represent a rogue flaw condition, one must rely on experienced judgment rather than on a reasonable knowledge of the applicable manufacturing quality. That approach, therefore, accepts some unknown risk of early life structural failures because the rogue flaws actually present in the structure might exceed the severity assumed for the *safe-life* calculations. Since earlier methods of fatigue analysis largely ignored the existence of rogue flaw occurrences, the extreme value approach represents a significant improvement. However, the *probabilistic approach* presented here provides a simple calculation procedure to assess the probable consequences of undetected rogue flaws no matter what their initial size.

The *safe-life* value calculated by the extreme value approach also depends on the accuracy of the growth rate computation for small cracks. One problem with accurately modeling the growth rate for small cracks involves the computation of the stress intensity at the tip of a corner crack located at a fastener hole (a common location for cracks to initiate). Whereas test results support the calculations for large cracks, it is much more difficult to obtain reliable test data for small corner cracks. Because the crack spends most of its life while it is small and slowly enlarging rather than when it is large and rapidly enlarging (because of the acceleration of the cracking process with crack length), errors in the consideration of the stress intensity for small cracks can have a significant effect on the calculated crack growth time. Thus, the calculation of the growth times for very small corner cracks must be considered to be much less precise than the calculations for cracks that are several times the thickness of the member. The probabilistic approach only requires crack growth rate information for the larger crack sizes at which element failures usually occur.

A further difficulty with an extreme value approach arises from the way the rogue load assumption interacts with judgments about the inspectability of the element postulated to contain the rogue flaw. Currently, the Air Force uses the limit load condition for the rogue load condition for all types of aircraft. The *limit load* is the maximum load that a given element would ever have to transmit provided that (1) the pilot always keeps the aircraft within the operating limitations specified by the manufacturer and (2) the neighboring elements continue to transfer their own loads. Because conditions can arise where an element may have to transmit a load larger than the limit load, *an element in an aircraft structure is usually designed to carry at least 150 percent of the limit load (assuming that the element has no cracks in it)*.

Although atmospheric turbulence can cause an aircraft to exceed a limit load condition, this very rarely occurs for a transport aircraft. Thus, if a transport continued to operate beyond the calculated *safe-life* number, the crack from the postulated rogue flaw condition would probably continue to grow before actual failure occurred. This might provide a significant opportunity to discover the crack before element failure occurred. Moreover, as the crack grows, it becomes increasingly more probable that the element failure will occur due to the application of a smaller load than limit load. The probabilistic approach provides a procedure that explicitly accounts for this factor.

#### Appendix C

##### THE POTENTIAL ROLE FOR THE PROBABILISTIC APPROACH

Throughout the life of an airframe structure, the manufacturer, operator, and any cognizant regulatory agency should continuously examine actions that might reduce the chances of a catastrophic structural failure. Although the kind of information generated by the probabilistic approach should interest each of these parties, the manufacturer would probably have to perform the actual calculations and analysis throughout the airframe's life cycle.

The motivation for developing the approach stems from the premises that a more efficient allocation of available resources during each aircraft's life cycle could reduce the probability of catastrophic structural failure. A joint application of both the fail-safe and the safe-life philosophies may do a better job of reducing the chances of a catastrophic structural failure than the application of only one philosophy. Thus the approach described here combines aspects of each philosophy in a procedure that either circumvents or deals with the major shortcomings of each philosophy. Although some design features of existing aircraft already reflect an ad hoc melding of philosophies, the approach proposed here formally combines elements of each philosophy. With this approach, one can rigorously assess the relative degrees of fail-safety associated with different elements forming an aircraft structure.

##### DESIGN PHASE

During this phase, the probabilistic approach could serve as (1) a design tool for helping to identify the most *failure-critical elements* by ranking elements according to the calculated probability that an undetected fatigue crack would lead to a structural failure; (2) a basis for revising design and manufacturing quality standards; and (3) a basis for providing the prospective operator a better opportunity to receive meaningful feedback on those design compromises that influence the possibility of a structural failure.

Although the manufacturer's legal liability provides a long-term incentive for him to minimize the possibility of a catastrophic structural failure, short-term incentives during the design phase can tend to work against fulfillment of that long-term responsibility. For example, a designer may have to add material to the structure to decrease stresses or to employ small multiple elements in place of a large single element. Such actions increase both weight and manufacturing costs while reducing the performance potential (e.g., payload), which can jeopardize the survival of a new aircraft design program, especially if the prospective operator does not understand the trade-off between the cost/performance factors and the possibility of catastrophic structural failure. The probabilistic approach generates information for the prospective operator to help him procure equipment that more efficiently minimizes the possibilities of catastrophic structural failures for a given set of *life-cycle resources*.

#### OPERATING PHASE

During this phase, this probabilistic approach could serve as (1) a tool for identifying failure-critical elements based on actual patterns of operational use; (2) a basis for establishing a maintenance and inspection standard or specification; and (3) a basis for providing the operator a better opportunity to understand the potential consequences of alternative combinations of operating plans, inspection programs, and structural rework/modification programs.

Because the actual patterns of operational use frequently deviate--sometimes significantly--from the assumed patterns used by the manufacturer when he designs the structure, the operator should consider requesting a reassessment of the failure-critical elements once he has established an actual pattern of operational use.

For the operator who has different aircraft types in his fleet, the probabilistic approach may provide information to help him standardize his inspection/rework/modification policies on a consistent basis. He can then allocate resources to minimize the chances that a catastrophic structural failure will occur to any of his different types of aircraft.

Although the shared legal liability of the operator and the manufacturer provides a continuing incentive for them to minimize the chances of a catastrophic structural failure, the operator and the manufacturer have divergent economic incentives during the operating phase. Rework and/or modification programs provide *profit* opportunities to the manufacturer and *expense* items to the operator. Some operators, therefore, try to closely examine the programs proposed by the manufacturer. However, a thorough examination of the details that underlie the rationale for the manufacturer's proposal can require an effort that few operators have the resources and will to undertake. In such cases, the operator can either accept the manufacturer's proposal, perhaps following a *pro forma* review, or he can negotiate a compromise program without fully appreciating the consequences of the compromises.

The disadvantageous position of the operator stems largely from limitations on the information available to him. For example, the operator may have:

- o The manufacturer's assessment of the time when fatigue cracks may develop.
- o A record of cracks detected in his fleet.
- o A sometimes ill-defined perception of the aircraft's fail-safe capability.

If the operator has no other information, he may not even negotiate the extent of the program proposed by the manufacturer. If of economic necessity he has to negotiate a more affordable program, he may not understand the possible consequences. The probabilistic approach can provide that understanding and thus a better assessment of the inspection/rework/modification programs proposed by the manufacturer.

Because the operator has a finite set of resources available for airframe maintenance, he cannot eliminate the possibility of a catastrophic structural failure. How effectively he reduces the probability of such an outcome depends on both the total resources available

*and the efficiency with which he applies those resources. Thus, a primary objective of this report is to propose an approach that can improve the quality and relevance of the information available to the operator.*

Appendix D

DEVELOPMENT OF INPUT INFORMATION

The input information used in the example came from the aircraft's manufacturer. This appendix provides further details on the preparation of that information.

LOAD INTENSITY DISTRIBUTION

To assemble this information, the manufacturer records and/or predicts the applied loads that the aircraft may experience during various operational uses. Using the frequency with which the aircraft experiences these uses, the manufacturer can project the expected distribution of applied loads that will occur over the aircraft's design lifetime and construct a corresponding distribution for the intensity of internal structural loads that occur at a specified location in the structure. With this kind of procedure, the manufacturer can construct an expected distribution for the maximum load intensity that occurs during each flight. He can also construct the expected peak-stress distribution that the fatigue and fracture analysts use.

Use of a peak-stress distribution, rather than a maximum load intensity distribution, may create some problems. Because development of the preferred distribution and development of the peak-stress distribution could occur simultaneously, and probably for a relatively small marginal cost, manufacturers should seriously consider development of the preferred distribution for the approach proposed here. Meanwhile, the peak-stress distribution used by the fatigue and fracture analyst can suffice for exploratory or illustrative applications.

A peak-stress distribution tends to cause an underestimate for the probability that the element will not fail, which tends to cause an overestimate in the probability that the structure will not fail. For example, examine the calculations for column 12 in Table 1 (see Sec. IV). Too low a value for column 11 causes too high a value for column 12; however, the same factor that causes the underestimate for

column 11 causes an overestimate for column 8. This causes an underestimate for the probability that the structure will not fail (see column 12). Thus, to an extent these errors offset each other.

Both errors stem from an overestimate of  $n(\sigma_{xf})$  that worsens as the value of  $\sigma_{xf}$  decreases. For large values of  $\sigma_{xf}$  the peak-stress distribution should not introduce very much error. However, as  $\sigma_{xf}$  decreases, the user should become increasingly more alert to the possibility of a problem.

In many situations, the indicated tendencies to overestimate or underestimate the failure probabilities will not seriously degrade the utility of the results. For example, if the user has made a consistent analysis of the comparative failure probabilities associated with the many different elements of a given structure, the relative ordering of the results should mostly remain valid, even though the absolute magnitude of individual results may deviate from the real probability. To make a consistent analysis, the same kind of distribution (i.e., the peak-stress distribution) should be used for each element. Two factors govern this distribution: location in the structure and a sequence of operating conditions. However, the second factor is the same for all elements in the same structure. Thus, when the second factor includes extraneous events (i.e., events not included in the maximum load intensity distribution), it does so for each element's peak-stress distribution; any tendency to overestimate the failure probabilities must apply to the estimate for *every* element. Thus, the *relative differences* in the failure probabilities will arise from differences in element location, element geometry, and element materials.

#### TIME REQUIRED FOR CRACK ENLARGEMENT

The operating time required for the postulated crack to enlarge by a specified amount depends on the material's crack growth characteristics, the operating environment, and the time-varying intensity of the stresses that occur around the crack's leading edge. The instantaneous value of the stress intensity ( $K$ ) depends on (1) the crack length; (2) the load intensity (e.g., the gross area stress); and

(3) a geometry dependent factor ( $\beta$ ) that relates the load intensity ( $\sigma$ ) to the stress intensity ( $K$ ) for a given crack length ( $a$ ). In general, to first approximation,

$$K = \beta\sqrt{\pi a} \sigma . \quad (D.1)$$

To determine the value of  $\beta$ , which often varies slowly with  $a$ , fracture mechanics analysts use stress analysis methods based on the linear theory of elasticity. Thus, as long as significant inelastic behavior does not occur, a rational scientific method exists for calculating the stress intensity.

#### Model of the Crack Enlargement Mechanism

For a single load cycle, laboratory experiments have shown that a relationship like

$$\Delta a = C(\Delta K)^m , \quad (D.2)$$

where to first approximation

$$\Delta K = \beta\sqrt{\pi a} \left( \sigma_{\max} - \sigma_{\min} \right) , \quad (D.3)$$

can predict the amount of crack enlargement ( $\Delta a$ ) caused by one load cycle that has a minimum load intensity,  $\sigma_{\min}$ , and a maximum load intensity,  $\sigma_{\max}$  (see Refs. 14-17). The empirical coefficients  $C$  and  $m$  reflect the material's crack growth characteristics. Researchers have found that the load ratio,  $\sigma_{\min}/\sigma_{\max}$ , and extreme values for the stress intensity range ( $\Delta K$ ) can influence these coefficients. However, for the aluminum alloys used in aircraft structures, and for the stress intensity ranges that such structures usually experience, the value of  $m$  usually remains in the range  $3 < m < 4$  (Ref. 16). In such cases, the amount of crack enlargement  $\Delta a$  approximately increases as the crack length is raised to a power between  $3/2$  and  $2$ . Thus, as the crack enlarges, the incremental amount of enlargement caused by one load cycle increases.

Model of the Load Fluctuations that Cause Crack Enlargement

The relevant portion of the structure will experience a variety of load intensity cycles having different values for  $\sigma_{\max}$  and  $\sigma_{\min}$ . Thus, to construct a representative sequence of cycles, the fracture mechanics analyst must first develop a model to represent the expected pattern of operational use. For an aircraft structure, he might use a block of 100 flights that includes a representative mix of different missions.

For each type of mission, he divides the flight into segments (e.g., taxi, takeoff, climb, cruise, etc.). For each segment, he uses expected values for the aircraft's operating parameters (e.g., weight, speed, altitude, flap configuration, engine thrust level, etc.) to calculate the expected applied loads and the expected intensity of the internal structural loads. He then calculates how this load intensity would change in response to aircraft maneuvers and atmospheric turbulence. To model the frequency and the extent of the expected maneuvers and atmospheric turbulence, the analyst often uses data obtained from analyses of load history recordings taken during flight. From such a model he constructs a sequence of load intensity cycles to represent the time-varying load intensity. After he repeats this exercise for each mission segment, he constructs a sequence of load intensity cycles that models the expected load intensity environment for a single mission. After completing the preceding exercise for each mission type, he constructs a sequence of load intensity cycles that models the expected load intensity environment for the 100 flights contained in his analysis block.

Model of the Crack Enlargement Process

For any initial crack size, the fracture mechanics analyst can use the 100-flight sequence of load intensity cycles to determine how much crack enlargement should occur because of 100 representative flights.

For example, where the stresses near the leading edge of the crack do not exceed the elastic limit, the analyst can take the initial crack length and the first load intensity cycle and compute the incremental

amount of crack enlargement caused by that cycle. He then continues this process for the remaining cycles. Alternatively, he can sometimes use the 100-flight sequence of load intensity cycles to experimentally determine the expected amount of crack enlargement. The viability of this option depends on the complexity of both the local geometry and the internal structural loads.

In either case, by reapplying the sequence, he can continue the process and determine the cumulative crack enlargement for 200 flights, 300 flights, ..., etc. He can use this information to plot the crack size versus the number of flights (or flight hours, as depicted in Fig. 4).

If significant plastic deformations occur (i.e., the elastic limit is exceeded) near the leading edge of the crack, the calculation method becomes somewhat more complicated. But the approach still parallels the preceding illustration (Ref. 17).

Note that the probability calculations in Table 1 (Sec. IV) start at a rather large initial crack length (0.7 inch). Because the calculation does not depend on the preceding crack propagation history, this probabilistic approach not only avoids the arbitrary assumption of an initial crack length, but it also avoids the problems peculiar to estimating the rate of crack enlargement for small corner cracks. Two kinds of problems arise. (1) It is difficult to accurately model the crack propagation behavior of a corner crack because it involves a three-dimensional geometry whereas the through-the-thickness crack only poses a two-dimensional geometry. (2) Many fastener systems that prolong fatigue life (crack initiation times) also retard crack enlargement rates for small cracks because of local inelastic behavior near the fastener hole. Such retardation, which Fig. 4 does not account for, only influences the crack propagation rate for the first few tenths of an inch. Thus, the probabilistic approach only uses the more reliable portion of the crack propagation curve.

Nonetheless, the use of a crack propagation curve, such as that in Fig. 4, involves certain implicit assumptions, such as:

- o Arrest of the crack propagation process does not occur.

- o The operational use of the structure approximates the assumed pattern used to develop the crack propagation curve.
- o The environment in which operational use occurs, especially humidity and salinity, approximates the environment of the laboratory tests used to determine the material's crack growth characteristics.

#### REMAINING STRENGTH OF THE CRACKED ELEMENT

The element will fail when  $K = K_c$ , where  $K_c$  is a property of the material (called the fracture toughness or the critical stress intensity). Thus, set  $K = K_c$  and  $\sigma = \sigma_{ef}$  in Eq. (D.1) to obtain

$$\sigma_{ef} = \frac{K_c}{\beta\sqrt{\pi a}}. \quad (D.4)$$

For the example application,  $\beta$  varies slowly with the crack length.

#### REMAINING STRENGTH OF THE STRUCTURE

The manufacturer can calculate the structure's remaining strength for the dual member failure case using methods similar to those used for the single member failure case. Although information for the dual member failure case is not available, the single member result, cited previously, can be used to estimate the dual member result. For example, to estimate the remaining strength for a double member failure, assume that (1) the second member has the same width (W) as the wider of the two members and (2) the members have comparable cross-sectional areas.

For a linear elastic material, the stress intensity around the crack's leading edge increases approximately as the square root of the crack length (Eq. (D.1)). Although the elastic characteristics of the joint where the members overlap will differ from the characteristics for the portion of the element between that joint and the adjacent joints, make the following assumptions:

- o The two panels of interest, together with their adjoining joints, have uniform elastic characteristics.

- o The stress intensity required for a panel failure to extend into an adjoining panel remains constant regardless of whether one or two members have already failed.

For the same stress intensity to occur after either a single or dual member failure,

$$\beta\sigma_{SPF}\sqrt{\pi W} = \beta\sigma_{DPF}\sqrt{\pi 2W} , \quad (D.5)$$

where  $\sigma_{SPF}$  is the minimum load intensity when single panel failure occurs and  $\sigma_{DPF}$  is the minimum load intensity when a dual panel failure occurs. Let  $\sigma_{SPF}^*$  (and  $\sigma_{DPF}^*$ ) be the load intensity level(s) that must be exceeded to fail the panel adjoining the single panel failure site (double panel failure site). Thus from the foregoing assumptions and Eq. (D.5),

$$\sigma_{DPF}^* = \sigma_{SPF}^*/\sqrt{2} . \quad (D.6)$$

For the example considered in this report,  $\sigma_{SPF}^* = 42$  ksi. Thus, for the example considered here, if the applied load that causes dual member failure has a load intensity (gross area stress) that exceeds approximately 30 ksi, this calculation suggests that the structure should fail.

Appendix E

HOW INPUTS INFLUENCE RESULTS

LOAD INTENSITY DISTRIBUTION

Figure 2 shows the expected number of times that the load intensity will exceed any given gross area stress level in one design lifetime of 30,000 flying hours. For example, the figure shows that the load intensity is expected to exceed the 42 Ksi gross area stress caused by a limit load condition once every 59 design lifetimes ( $1/0.017 = 59$ ), whereas it is expected to exceed 67 percent of the limit load gross area stress (28 Ksi) once every design lifetime. Thus, as the gross area stress increases from 67 to 100 percent, the probability in Table E.1 decreases by a factor of 59. Because a modest change in the gross area stress can cause a substantial change in that probability, the maximum load intensity distribution has a significant influence on the results calculated by this methodology.

Table E.1

INFLUENCE OF LOAD INTENSITY DISTRIBUTION  
ON THE EXAMPLE CALCULATION

Load Intensity as a Percent of the Limit Load Gross Area Stress	Probability that During a Single 5-Hour Flight the Load Intensity Will Exceed the Level in Column 1
67	$\frac{5}{1 \times 30,000} = .00017$
100	$\frac{5}{59 \times 30,000} = .0000028$

SOURCE: Fig. 2.

REMAINING STRENGTH OF THE CRACKED ELEMENT

Figure 3 shows that once the crack has broken through the ligament between the fastener hole and free edge of the element ( $a = 0.655$  inch),

the remaining strength begins to decrease with increasing crack length. The fracture mechanics analysis, from which this figure came, assumed that once the crack broke through the ligament, the crack continued to develop from a corner on the opposite side of the fastener hole. This corner crack had an assumed shape of a quarter circle. Once it reached a radius that equaled the thickness of the element (0.25 inch), the analysis assumed that the crack became a through-the-thickness crack; at that point,  $a = 0.905$  inch (see Fig. 4, also).

Observe from Fig. 3 that once the crack has reached a surface length of about 0.8 inch, the limit load condition (42 ksi) will cause the element to fail. Similarly, once the crack has reached a surface length of about 1.3 inches, element failure will occur at a load condition that causes a gross area stress of 67 percent of the limit load gross area stress (28 ksi). Thus, for this example, a half-inch enlargement in crack length (0.8 to 1.3 inches) has reduced the remaining strength from 100 to 67 percent of the limit load gross area stress. Adding this information to Table E.1 shows that (see Table E.2) if the element has not failed during a prior flight, the probability that element failure will occur during the next 5-hour flight increases by a factor of 59 because of this half-inch enlargement in crack length.

#### TIME REQUIRED FOR CRACK ENLARGEMENT

Figure 4 shows that it would take about 700 flying hours for a 0.8 inch crack to enlarge to 1.3 inches. Adding this information to Table E.2 and rearranging the columns shows (see Table E.3) that if the element has not failed during a prior flight, the probability that the element failure will occur during the next 5-hour flight increases by a factor of 59 because of the half-inch enlargement in crack length that occurs over the period of 700 flight hours.

Because the probability increases so rapidly, the calculation procedure must use a calculation step size much smaller than 700 flight hours. Moreover, the procedure must also compute the cumulative probability that the element has failed from  $t = 0$  (the beginning of the calculation) to some later time of interest. Section III provides the details for that calculation.

Table E.2  
INFLUENCE OF CRACK LENGTH ON THE EXAMPLE CALCULATION

Load Intensity as a Percent of the Limit Load Gross Area Stress	Crack Length at Which the Load Intensity in Column 1 will Fail the Element	Probability that During a Single 5-Hour Flight the Load Intensity Will Exceed the Level in Column 1
100	0.8	$\frac{5}{59 \times 30,000} = .0000028$
67	1.3	$\frac{5}{1 \times 30,000} = .00017$

SOURCE: Fig. 3 Fig. 2

Table E.3  
INFLUENCE OF ELEMENT'S REMAINING STRENGTH  
ON THE EXAMPLE CALCULATION

Time in Flight Hrs (use lower scale in Fig. 4)	Crack Size (surface length in inches)	If the Load Intensity Exceeds This Level, the Element Fails (percent of the limit load gross area stress)	Probability that During a Single 5- Hour Flight, the Load Intensity Will Exceed the Level in Column 3
300	0.8	100	$\frac{5}{59 \times 30,000} = .0000028$
1000	1.3	67	$\frac{5}{1 \times 30,000} = .00017$

SOURCE: Fig. 4 Fig. 3 Fig. 2

REMAINING STRENGTH OF THE STRUCTURE

Figure 2 shows that, once every 1.7 design lifetimes, the peak stress exceeds 30 ksi. Adding this information to Table E.3 shows (see Table E.4) that (1) if the postulated crack reaches an 0.8 inch length without causing the element to fail, then during the next flight the probability of element failure equals the probability that element failure will cause structural failure; and (2) if the postulated crack reaches a 1.3 inch length without causing the element to fail, then during the next flight, the probability of element failure equals 1.7 times the probability that element failure will cause

Table E.4  
INFLUENCE OF STRUCTURE'S REMAINING STRENGTH  
ON THE EXAMPLE CALCULATION

Time In Flight Hours (use lower scale in Fig. 4)	Crack Size (surface length in inches)	If the Load Intensity (Percent of the limit Load Gross Area Stress) Exceeds This Level	The Dual Panel Ele- ment Fails	The Struc- ture Fails	The Element Fails	Probability that During a Single 5-Hr Flight the Load Intensity Will Exceed the Level at which The Struc- ture Fails
300	0.8	100	100	100	.0000028 <sup>a</sup>	.0000028 <sup>a</sup>
1000	1.3	67	71	71	.00017 <sup>b</sup>	.00010 <sup>c</sup>

SOURCE: Fig. 4 Fig. 3 -<sup>d</sup> Fig. 2 Fig. 2

<sup>a</sup> 
$$\frac{5}{59 \times 30,000} = .0000028.$$

<sup>b</sup> 
$$\frac{5}{1 \times 30,000} = .00017.$$

<sup>c</sup> 
$$\frac{5}{1.7 \times 30,000} = .00010.$$

<sup>d</sup> See preceding discussion.

structural failure. Note that this element failure probability exceeds the structural failure probability for all crack lengths greater than the *transition length* (see Fig. 1). Also note that the element failure probability equals the indicated structural failure probability for all crack lengths smaller than the transition length.

Figure 1 illustrates why this transition behavior occurs. For cracks smaller than the *transition length*, the load intensity at which element failure occurs always exceeds the level at which structural failure attends any element failure; compare the short-dash curve to the long-dash curve in Fig. 1. For cracks larger than this transition length, the load intensity at which structural failure occurs always exceeds the level at which element failure occurs. Thus, beyond this transition crack length, the element may fail without failure of the structure.

#### THE CALCULATION PROCEDURE

Each row in Table E.4 illustrates a basic calculation procedure that recurs during a corresponding phase of the calculation. During the first phase, the crack's length has not reached the transition length; see the first row in Table E.4. During the second phase, the crack's length has surpassed the transition length; see the second row in Table E.4. During either phase, the probabilities depicted in Table E.4 will vary rapidly with the crack length, which in turn varies with time. Moreover, to calculate the cumulative probabilities over time, one must first develop the probabilities in Table E.4 as a function of time (or, in other words, crack length). Thus, the basic calculations illustrated in Table E.4 must recur for successive increments of time. Section III presents the complete procedure for calculating the cumulative probabilities.

Appendix F

THE ASSUMPTIONS AND SOME IMPLICATIONS

Assumption 1: Detection of the cracked element does not occur before the crack fails the element.

In some situations, the probability of element failure will remain quite small as the crack continues to enlarge past the size at which crack detection begins to be a more probable event than element failure. For example, consider a through-the-thickness crack that has enlarged to a size that measures several inches along the surface of an element. If the element forms part of the wing's lower surface, it may also form the lower surface for an integral fuel tank. In the past, fuel leakage has frequently indicated the presence of a crack; either the flight crew has recognized the loss of fuel while in flight or the maintenance crew has recognized stains on the wing. Thus, to properly analyze the results obtained from applying this approach to a specific element, one should carefully evaluate the implications of Assumption 1. To facilitate such an *ex post facto* evaluation, the calculation procedure provides the probability that the element will not fail as a function of crack length.

Assumption 2: Once the cracked element fails, a completely effective repair or replacement of that element occurs before the structure begins the next operating interval.

If the failed element forms part of the structure that the flight crew cannot readily observe during a pre-flight walkaround inspection (e.g., the upper surface of the horizontal tail), that inspection cannot be relied on to satisfy Assumption 2. However, if the maintenance crew performs an inspection of that area prior to every flight, then that action might plausibly satisfy the assumption. Although an analysis should consider the possibility that the crew might not detect the

element failure, the current approach separates such considerations from the basic calculation.

Of course, if the maintenance crew cannot observe the candidate element because another element blocks it from view, then external inspection will not satisfy Assumption 2. For example, both the 707 and L-382 accidents described in Appendix A involved the failure of a *spar cap*. A *cap* in a wing or tail structure connects the external skin to the internal webs that connect the upper and lower surfaces; a *spar cap* is a cap that runs along the span of the wing (or tail). In some aircraft structures, maintenance crews cannot visually inspect the spar cap without opening up the structure because the external skin of the structure completely obscures the spar cap from view. In other aircraft, immediate detection of a failed spar cap may not occur if it only has a small externally visible width. Thus, in any application of this methodology, one must carefully examine the postflight observability of the postulated element failure.

In many cases, however, maintenance crews and flight crews can observe many of the fatigue prone elements. For example, the USAF KC-135 tanker force has survived thirty incidents where a skin panel on the wing's lower surface has either failed in flight or has developed a crack with a surface length of several inches (Ref. 18). The C-130 and the C-141A forces have also survived such incidents on several occasions. If an element cannot satisfy Assumption 2, that fact alone raises serious questions about the degree of fail-safety associated with that element.

Assumption 3: All operating intervals have the same duration.

For the aircraft considered in the example, most flights have a duration of from 4 to 6 hours; thus, the report uses an assumed operating interval of 5 hours. Deviations from that assumed average should not significantly influence the calculated result as long as each operating interval has the same order of magnitude and the duration of the largest interval remains small in comparison with the duration of the calculation time frame.

Assumption 4: When each operating interval starts, a single step increase in the crack length provides an adequate representation for the total crack extention that occurs during the operating interval.

As long as the length of the operating interval remains small in comparison to the time required for the probability of no element failure to decline (e.g., see column 10 in Table 1), this assumption should not cause any significant difficulties.

Assumption 5: The operations during each operating interval occur independently of the operations during all previous operating intervals.

This assumption should remain valid as long as the mission for each operating interval is assigned independently of the previous missions. Thus, for example, this assumption should not pose a problem as long as each aircraft experiences a representative mixture of missions.

Assumption 6: If the load intensity exceeds the level required to fail the element more than once during a given operating interval, the highest load intensity level occurs first.

If Assumption 6 does not hold and if the element failure induces a dynamic response from the structure that causes a significant amplification of load intensity, then in certain situations a favorable order of occurrence can reduce the chances that a structural failure would occur. For example, consider two separate events with applied load levels ( $L_1$  and  $L_2$ ) that exceed the level required to fail the element. Assume that event 1 ( $L_1$ ) does not fail the structure and event 2 ( $L_2$ ) fails the structure *only if* it also fails the element. Thus, in this situation, if the element fails prior to the occurrence of event 2, then the dynamic response of the structure to that failure

cannot amplify the internal structural loads caused by  $L_2$ . Without such dynamic amplification,  $L_2$  cannot exceed the structural failure level. Therefore, if event 1 occurs before event 2 occurs, event 2 cannot fail the structure. However, if event 2 occurs before event 1 would have occurred, then the structure does fail. Thus, by assuming that  $L_2$  occurs first (Assumption 6), the user makes a worst case assumption. However, from a practical standpoint, such an assumption should not seriously influence the result, especially for aircraft structures, where the element failures do not usually cause a large enough dynamic amplification for the favorable sequence (event 1 followed by event 2) to occur very frequently. In any case, with respect to Assumption 6, this approach provides a conservative assessment of the structural failure probability.

Appendix G  
AN ALTERNATIVE FORM FOR THE EQUATIONS

For  $x_i \ll 1$ ,

$$\prod_{i=1}^j (1 - x_i) = \left[ 1 - \frac{1}{2} \sum_{i=1}^j x_i^2 e^{x_i} \right] \exp \left\{ - \sum_{i=1}^j x_i \right\} + \text{Higher Order Terms.}$$

Thus, because  $r_{ef}(t_j) \Delta t \ll 1$ , rewrite Eq. (7a) as, approximately

$$P_{nef}(0, t) = f(0, t) \exp \left\{ - \int_0^t r_{ef}(\tau) d\tau \right\}, \quad (G.1a)$$

where

$$f(0, t) = 1 - \frac{1}{2 \Delta t} \int_0^t \left\{ r_{ef}(\tau) \Delta t \right\}^2 \exp \left\{ r_{ef}(\tau) \Delta t \right\} d\tau. \quad (G.1b)$$

However,  $f(0, t) \approx 1$  whenever  $r_{ef}(\tau) \Delta t \ll 1$  or  $\Delta t \rightarrow 0$ .

From Eq. (4)

$$r_{ef}(\tau) = n(\sigma_{ef}(a(\tau))) / t_d. \quad (G.2)$$

From Eqs. (4), (7a), and (7b)

$$P_{nsf}(0, t) = 1 - \int_0^{\tilde{t}} \frac{P_{nef}(0, \tau)}{1 - r_{ef}(\tau) \Delta t} r_{ef}(\tau) d\tau - \frac{n(\sigma_{rs})}{t_d} \int_{\tilde{t}}^t \frac{P_{nef}(0, \tau)}{1 - r_{ef}(\tau) \Delta t} d\tau, \quad (G.3)$$

where  $\tilde{t}$  is defined by

$$\sigma_{ef}(\tilde{t}) = \sigma_{rs} . \quad (G.4)$$

However,  $1 - r_{ef}(\tau) \Delta t \approx 1$  whenever  $r_{ef}(\tau) \Delta t \ll 1$ .

Appendix H

EXAMPLE CALCULATION FOR A SINGLE PANEL FAILURE

Table H.1 considers a variation of the example presented in Table 1. Whereas Table 1 considered the dual panel failure mode, with a residual strength of  $\sigma_{rs} = 30$  ksi, Table H.1 considers the single panel failure mode where  $\sigma_{rs} = 42$  ksi. Except for  $\sigma_{rs}$ , the inputs have not changed; thus the values in columns 9 and 11 have simply been copied from Table 1. To facilitate such transcriptions and comparisons, Table H.1 uses the same column numbers as Table 1.

The results in column 12 show that, for the significant digits displayed in column 13, the first interval contributes to the calculated value for  $P_{nsf}$ . Thus, in theory, the calculation should commence with a crack length smaller than 0.74 inch (see Table 1). A length of 0.65 inch would suffice. For example, redefine the origin for the time  $t$  such that  $a(0) = 0.65$  inch, and repeat the calculations in Tables 1 and 2. The last value in Table 2 for  $P_{nsf}$  would change from 0.999018 to 0.998933. Thus, to three significant digits, Table H.1 provides a result that is insensitive to whether a smaller initial crack length is used to start the calculation.

Table H.1  
EXAMPLE CALCULATION FOR A SINGLE PANEL FAILURE

From Table 1		Eq. (6)		Values From Table 1		Probability of No Structural Failure [Eq. (10b)]	
$\alpha$	$k_\alpha$	$r_{sf}(\bar{t}_\alpha) \Delta t$ = $\min \left( \frac{7}{n(\sigma_{rs}) \frac{\Delta t}{t_d}} \right)$	$\bar{t}_\alpha$	$P_{nef}(0, \bar{t}_\alpha)$	$\bar{t}_\alpha$	$P_{nsf}(0, \bar{t}_\alpha) =$ $\frac{(2)(8)(11)}{(9)} - (12)_{\alpha-1}$ - $(12)_\alpha$	
(1)	(2)	(8) <sup>a</sup>	(9)	(11)	(12)	(13)	
1	20	$9.58 \times 10^{-7}$	1.0000	1.0000	.000019	.999981	
2	20	$1.60 \times 10^{-6}$	1.0000	1.0000	.000032	.999930	
3	20	$2.83 \times 10^{-6}$	1.0000	.9997	.000057	.999841	
4	20	$2.83 \times 10^{-6}$	.9999	.9981	.000056	.999728	
5	20	$2.83 \times 10^{-6}$	.9999	.9947	.000056	.999616	
6	20	$2.83 \times 10^{-6}$	.9998	.9877	.000056	.999504	
7	20	$2.83 \times 10^{-6}$	.9994	.9700	.000055	.999393	
8	10	$2.83 \times 10^{-6}$	.9989	.9474	.000027	.999311	
9	10	$2.83 \times 10^{-6}$	.9983	.9214	.000026	.999258	
10	10	$2.83 \times 10^{-6}$	.9971	.8794	.000025	.999207	
11	10	$2.83 \times 10^{-6}$	.9952	.8134	.000023	.999159	
12	10	$2.83 \times 10^{-6}$	.9922	.7163	.000020	.999116	
13	10	$2.83 \times 10^{-6}$	.9867	.5790	.000017	.999079	
14	10	$2.83 \times 10^{-6}$	.9772	.4019	.000012	.999050	
15	5	$2.83 \times 10^{-6}$	.9672	.2700	.000004	.999034	
16	5	$2.83 \times 10^{-6}$	.9583	.1847	.000003	.999027	
17	5	$2.83 \times 10^{-6}$	.9375	.1081	.000002	.999022	
18	5	$2.83 \times 10^{-6}$	.897	.0454	.000001	.999019	
19	5	$2.83 \times 10^{-6}$	.853	.0119	.000000	.999018	
20	5	$2.83 \times 10^{-6}$	.767	.0014	.000000	.999018	

<sup>a</sup>Column numbers refer to Table 1.

Appendix I

SUGGESTIONS FOR REFINING THE EXAMPLE CALCULATIONS

The example calculations in Sec. IV were intended primarily to be illustrative. To refine the calculations for the aircraft used in the example, one should have the following information:

1. A revised load intensity curve (Fig. 2) based on
  - a. maximum peak-stress per flight data, and
  - b. the current aircraft configuration (i.e., include the load reducing influence of the current load distribution control system).
2. For the cracked element, a revised remaining strength curve (Fig. 3) that accounts for the influence of integral stiffeners.
3. A revised crack growth curve (Fig. 4) that
  - a. accounts for the influence of the current load distribution system,
  - b. accounts for the influence of stiffeners, and
  - c. pertains to the precise geometry of the most critical component at the service limiting joint.
4. A more precise calculation of the structure's residual strength for the postulated dual panel failure condition.
5. Items 1 and 3 above for a mission mix containing only
  - a. training missions, and
  - b. high payload logistics missions.

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